Bootstrapping vs. Asymptotic Theory in Property and Casualty Loss Reserving

The Honors Program
Senior Capstone Project
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April 2015

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ABSTRACT

One of the key functions of a property and casualty (P&C) insurance company is loss reserving, which calculates how much money the company should retain in order to pay out future claims. Most P&C insurance companies use non-stochastic (non-random) methods to estimate these future liabilities. However, future loss data can also be projected using generalized linear models (GLMs) and stochastic simulation. Two simulation methods that will be the focus of this project are: bootstrapping methodology, which resamples the original loss data (creating pseudo-data in the process) and fits the GLM parameters based on the new data to estimate the sampling distribution of the reserve estimates; and asymptotic theory, which resamples only the GLM parameters (fitted from an original set of data) from a multivariate normal distribution to estimate the sampling distribution of the reserve estimates. Using Excel, R, and SAS software, the copulas of the GLM parameter estimates from the stochastic methods will be compared to the copula from a multivariate normal distribution. Ultimately, the Value at Risk (VaR) and Tail Value at Risk (TVaR) results from each method's sampling distribution will be compared to each other, with the goal of showing that the two methods produce significantly different reserve estimates and risk capital estimates at the low end of the reserve distribution. This would answer the question as to whether the asymptotic theory procedure sufficiently approximates real-world scenarios.

INTRODUCTION

Property and casualty (P&C) insurance is one of the major forms of insurance available in today's market (the others being life insurance and health insurance). However, P&C insurance covers different risks than the other two: this type of risk transfer protects against losses faced by homeowners and business owners. Exposures protected include automobiles, houses, buildings, valuable items, and different types of liabilities.

The two major tasks faced by actuaries who work in P&C insurance companies are ratemaking and loss reserving. Ratemaking is the pricing of insurance policies, which is the process of establishing the amount of premium to charge each customer in order to adequately cover losses, expenses, and a profit load (on a pooled risk basis). Loss reserving is the estimation of how much money the insurer will need to hold to cover future reported losses. The process of loss reserving involves using the upper half of the loss reserving triangle, which consists of loss data previously reported to the company (shaded light gray in the exhibit below) to project loss amounts in the lower half of the triangle (shaded dark gray in the exhibit below). Insurance companies strive to estimate reserve amounts as accurately as possible because over-reserving would hinder the companies' use of the capital for investing, while under-reserving would weaken their capacity to withstand catastrophic events (due to a lower amount of risk capital held). Most insurance companies do not utilize stochastic methodologies to predict their loss reserves; rather, they use point estimates, which do not quantify uncertainty like stochastic models do. Some of the popular methods used, as outlined by Friedland (2010), include the chain ladder technique and the Bornhuetter-Ferguson method. While there is currently no industry consensus on the use of stochastic models, these models do provide quantitative measures that can assist company management in determining efficient levels of risk capital for specific lines of business, or for the company as a whole.

Period	Development Year	1	2	3	4	5	6	7	8	9	10
Policy Year											
1		463,742	573,866	710,142	878,779	613,068	427,698	298,378	208,159	145,219	101,310
2		487,019	602,671	745,787	922,888	643,840	449,166	313,355	218,607	152,508	106,395
3		511,464	632,921	783,221	969,212	676,157	471,711	329,083	229,580	160,163	111,736
4		537,136	664,690	822,533	1,017,860	710,096	495,388	345,601	241,104	168,203	117,344
5		564,097	698,053	863,819	1,068,950	745,738	520,254	362,948	253,205	176,645	123,234
6		592,411	733,091	907,177	1,122,604	783,169	546,367	381,165	265,915	185,512	129,420
7		622,146	769,887	952,712	1,178,952	822,479	573,791	400,297	279,262	194,823	135,916
8		653,374	808,530	1,000,532	1,238,128	863,762	602,592	420,390	293,279	204,602	142,738
9		531,024	657,126	813,174	1,006,278	702,016	489,752	341,668	238,360	166,289	116,009
10		431,586	534,074	660,900	817,844	570,557	398,041	277,688	193,725	135,150	94,285

Figure 1 – Sample Loss Development Triangle

LITERATURE REVIEW

McCullagh and Nelder (1989) wrote the foundational book on generalized linear models, as it describes such topics as the origins of GLMs and how to calculate residuals. The paper written by Anderson et al. (2007) acts a simplified reference guide to the basic definitions of each part of a GLM, in addition to providing illustrative examples on how to analyze error structures, which are built into the GLMs themselves. Hartl's conference presentation (2013) also provides an illustrative example of some of the principles described in the Anderson paper.

While Barnett and Zehnwirth (2007) describe a lognormal model (not a GLM), this is still an influential paper for the field of stochastic reserving for P&C development triangles. The authors demonstrate, with statistical goodness-of-fit tests, why traditional loss development methods are not a good model for most data sets.

Davison and Hinkley (1997) break down the process of applying bootstrapping to GLMs with concrete real-world examples. Pinhiero, Andrade e Silva, and Centeno (2003) explore bootstrapping in an applied manner with the specific insurance example of loss reserving. England and Verrall (2006) reinforce the general concept of bootstrapping, with its benefits and limitations, but explain other necessary material. The chain-ladder technique is explored and contrasted against stochastic reserving practices. Wüthrich and Merz (2008) also detail how to apply GLMs and bootstrapping practices to insurance examples. They touch upon the GLMs from the exponential dispersion family and parametric bootstrapping. They also detail a general claim-handling process for non-life insurance claims, which establishes a mind-frame regarding how claims are documented and processed. Hartl (2010) provides a specific framework for this project with his paper on bootstrapping, GLMs, and deviance residuals.

Asymptotic theory is explored in a traditional statistical manner in the book by Lehmann and Casella (1998). Alai and Wüthrich (2009) explain asymptotic theory in a more applied, actuarial context, in which, as the number of data points increases, the difference between the simulated parameter estimates and the "true" parameter estimates becomes approximately normally distributed, with mean zero and a Fisher information matrix describing the variance-covariance structure.

According to the asymptotic property of maximum likelihood estimation (for large data samples), the parameters estimates in the linear predictor are bias-free. However, this does not mean that the exponential of the linear predictor is also bias-free. Kosmidis (2014) explains how bias appears and how to adjust for cases of its existence in small data samples.

Risk capital and the process of risk modeling are well-defined in the P&C insurance industry. Insurers must have a method to calculate how much extra capital they should retain in case of a rare event. Rech et al. (2012) provides a comprehensive guide to risk modeling in the P&C industry.

METHODOLOGY – TECHNICAL NOTES

The GLM that will be used in this study is an exponential model (with a logarithm link function) and an over-dispersed Poisson variance structure. The over-dispersion refers to the presence of a dispersion parameter, which is explained in the Lecture 25 paper used by Professor Rachel Altman.

Barnett and Zehnwirth (2007) introduce the PTF class of lognormal regression models for development triangles. This class' design matrix is very similar to what will be used in this project. The PTF class also includes models that include the payment year dimension in the analysis, which is important for this study.

The output that is generated from the Excel models that will be used is produced by using a method called bootstrapping. Bootstrapping is a Monte Carlo simulation technique based on repeatedly applying an estimator to randomly generated sets of pseudo data. An in-depth discussion of this process is the one by Pinhiero, Andrade e Silva, and Centeno (2003). They break down the process into key components: fitting the GLM to the existing data, producing fitted data for the upper half of the reserving triangle, creating forecasted reserve numbers for the bottom half of the triangle, rescaling the residuals from the upper half of the triangle and resampling them with replacement, creating pseudo-data from the resampled residuals for the upper half of the triangle, and then repeating the process over again for a specified number of bootstrapped estimates.

A few different residual resampling methods for the GLM will be used in this project. These methods are utilized to ensure that negative loss data is not modeled (it is not possible to take the logarithm of a negative number). To protect against this occurrence, Hartl (2014) formulated two different procedures to alter the residuals so that reserve figures would not drop below zero. The first is using a shifted Limited Pareto distribution instead of using the scaled residuals from the model. This parametric resampling method draws values from a distribution which has a similar mean-variance relationship to the model being used for this project. The second method is Split Linear Rescaling, which splits the residual pool into lower and higher groups if residuals are a certain percentage below the mean. The values in the lower group are “squeezed” together to avoid negative numbers, which preserves the mean but alters the variance. To counterbalance this effect, values in the higher group are “expanded,” preserving the mean, while offsetting the variance change in the lower group.

To have something to compare the bootstrapped parameter estimates and sampling distribution of reserve estimates to, a closed-form expression of a multivariate normal distribution is needed for the asymptotic theory approach; Genz (1992) provides that in his paper. The Gaussian copula can then be computed using such techniques as Hothorn, Bretz, and Genz (2001) describe for R statistical software (the code used can be found in Appendix A). This copula will be used to sample the GLM parameters from, instead of creating pseudo data like the bootstrapping procedure does.

The tail risk measures that will be compared are Value at Risk (VaR) and Tail Value at Risk (TVaR). While VaR and TVaR of higher, right-tailed percentiles (greater than or equal to ninety-nine percent) are important in evaluating whether a model over-reserves, more attention will be given to VaR and TVaR of lower, left-tailed percentiles (less than or equal to one percent) because under-reserving creates more of an issue with insurer solvency. Due to that focus, a slightly different calculation of TVaR will be used: $TVaR_p(X)$ = average of all values less than the p^{th} percentile of the sampling distribution.

METHODOLOGY – PROCESS

Parameter Analysis

The goal of the parameter analysis procedure was to show that the copulas of the GLM parameters from each of the bootstrapping sampling methods were statistically significantly different than the copula of the GLM parameters from the multivariate normal distribution.

The bootstrapping model in Microsoft Excel was ran for ten million iterations per resampling method (Limited Pareto and Split Linear Rescaling). The output from each of these two methods was exported as a CSV file, with each file having seven columns of data. The first six columns held the simulated values of each of the six parameters used in the model; the seventh column held the reserve residual for each iteration (the difference between modeled reserve and the actual reserve). Each CSV file was then processed in SAS 9.3 using the code found in Appendix B. The Limited Pareto CSV file was first uploaded into SAS. For each parameter, a number from zero to nine was assigned to the estimate from each iteration. The number reflected the decile that each estimate fell into, with respect to the complete list of the ten million estimates for that specific parameter. For example, zero represented the first decile, one represented the second decile, and so on. After each parameter estimate was assigned an identifier, each iteration of the six parameters underwent a transformation in order to establish a single identifier that could be used as a comparison figure. The decile identifier for the first parameter was multiplied by 100,000, the decile for the second parameter was multiplied by 10,000, and so on, ending with the sixth parameter being multiplied by one. The sum of these six numbers for each iteration was taken, and an identifier, with a range of zero to 999,999, was created. In effect, this created a six-dimensional copula (a “hypercube”) that displayed the characteristics of the entire six-parameter structure, with the numerical identifier acting as a binning value. A simple count of the number of iterations belonging to each bin was then performed. The output from this step was exported as a CSV file, and the entire process was then repeated for the Split Linear Rescaling CSV file.

The ten million sets of parameter estimates derived for the asymptotic approximation were calculated using a multivariate normal distribution in R. In addition, a similar process to the SAS code was applied to the R output in order to establish bin identifiers for each set of parameter estimates and sums of the probabilities (rather than counts) of each bin identifier. However, in order to conform to Chi-squared statistic conventions, there needed to be restrictions on the totals in each bin for each unique identifier, namely, a minimum of one hundred. In order to accomplish this, an additional step was taken to order the parameters from smallest to largest and to regroup them into new bins with minimum value of one hundred. The total number of bins remaining after this step was 68,405. The output from this procedure was pasted into two new Excel workbooks (one for comparison to the Limited Pareto method and the other for comparison to the Split Linear Rescaling method). Since the total probabilities of all the bins did not add precisely to 1.00 (0.999995608 to be exact), all of the probabilities were divided by this total in order to make their sum exactly 1.00. To transform the probabilities into counts, each probability was multiplied by ten million.

In one of the newly created workbooks, the Limited Pareto CSV file data was pasted into a new worksheet. An Excel VLOOKUP was used to map the Limited Pareto data to the bins that were defined by the multivariate normal parameter sorting, and then the Limited Pareto data was summed for each bin. Chi-squared statistics (of the form $(Observed - Expected)^2 / Expected)$ were calculated for the bin totals, with the Limited Pareto counts as the observed and the multivariate normal counts as the expected. The statistics were then added, and using the Excel CHISQ.DIST function, the left-tailed Chi-squared p-value was calculated. The same process was repeated for the Split Linear Rescaling data in a separate workbook.

<u>Reserve Estimate Analysis</u>

The goal of the reserve estimate analysis procedure was to show that the tail measures of risk of the sampling distributions of reserve estimates calculated using bootstrapping methods were significantly different than those calculated using the asymptotic theory approximation, in terms of risk capital needed.

Before beginning the analysis of reserve estimates, a few adjustments needed to be made in the VBA code in the Excel model. As stated before, the parameters in the linear predictor were assumed to be bias-free, but the exponential of the linear predictor could not fall under the same assumption. This could be seen when the reserve estimate for the fitted model was compared to the average of the bootstrapped simulations of the reserve estimate: the averages of the bootstrapped estimates were consistently higher than the fitted estimates. To compensate for the bias, the code inserted into the model not only gave the reserve estimates for the non-bias-adjusted model, but also included two additional columns of reserve estimates: one for the reserve estimates calculated when the model was adjusted with an arithmetic correction factor, and the other for the reserve estimates when the model was adjusted with a multiplicative correction factor. In each case, the triangle was fitted with modeled loss figures, and the bias in each cell was noted and kept track of. Once the model fitting was completed, the additive adjustment subtracted out the accumulated bias from each cell, while the multiplicative adjustment multiplied each unadjusted cell by the factor *Projected Reserve / (Projected Reserve + Bias)*.

The adjusted bootstrapping model was then run for 100,000 iterations, producing 100,000 reserve estimates for each combinations of the three model characteristics: resampling method, number of diagonals used to fit the GLM, and number of payment period parameters used in the model. The three resampling methods used were the Limited Pareto, Split Linear Rescaling, and the multivariate normal distribution. The GLM was either fitted using the loss data from the lower five or all ten diagonals of the upper triangle. Also, the GLM had either two payment period parameters (equivalent to one parameter plus a constant offset value) or one payment period parameter (equivalent to no parameter plus a constant offset value). Each of the twelve model combinations was run on five different triangles (Taylor and Ashe, Alaska Workers Compensation, Chubb Personal Auto Liability, Chubb Commercial Multiple Peril, and ACE 2013 General Liability, which are all included in Appendix E), for a total of sixty CSV files of sampling distributions of reserve estimates. Each file contained the sampling distribution for the non-bias-adjusted reserves, additively-adjusted reserves, and multiplicatively-adjusted reserves.

Since the focus of the study was focused more on under-reserving than over-reserving, the left tails of the sampling distributions were analyzed, at 0.40%, 1%, and 5%. VaR and TVaR statistics for each of the additively-adjusted and multiplicatively-adjusted sampling distributions were calculated. The VaR figures were calculated by using the LARGE Excel function in order to find the (1-p%)*100,000 largest estimate in the sampling distribution. The TVaR figures were calculated by using the AVERAGEIF Excel function to take the average of all of the estimates smaller than the VaR number at the corresponding percentile. The data was then regrouped into five different Excel workbooks, one for each loss triangle used, and then partitioned by bias adjustment method, resampling technique, number of diagonals used, and number of payment period parameters used. The tail measures of risk were expressed as percentages of the projected reserve from their corresponding models (number of diagonals and number of payment period parameters used).

The endgame of analyzing the tail measures of risk of the sampling distributions of the reserve estimates was to examine the differences between the three methods in terms of the risk capital needed. This was achieved by creating one more set of calculations: comparing both the Limited Pareto and Split Linear Rescaling percentage differences to the percentage differences from the multivariate normal method. Each VaR and TVaR percentage statistic from the bootstrapping methods was divided by its counterpart from the asymptotic approximation VaR and TVaR statistics. In effect, this calculation showed the ratio of risk capital needed by each bootstrapping method in relation to the asymptotic theory approximation.

<u>RESULTS</u>

<u>Parameter Analysis</u>

The sum of the Chi-squared statistics for the Limited Pareto resampling method equaled 827,185,458.31. With 68,404 degrees of freedom, the left-tailed Chi-squared probability was calculated in Excel to be 1.00, which meant that the right-tailed p-value equaled approximately zero (the exact probability was too small for Excel to display). This meant that the two resampling methods produced very highly significantly different parameter copulas.

The same process above was repeated for the Split Linear Rescaling Excel workbook. The sum of those statistics was 169,069.33. With 68,404 degrees of freedom, the left-tailed Chi-squared probability was calculated in Excel to be 1.00, which meant that the right-tailed p-value equaled approximately zero (the exact probability was too small for Excel to display). This meant that the two resampling methods produced very highly significantly different parameter copulas.

Quantile-quantile (Q-Q) plots were created in SAS Enterprise Guide 9.3 for each parameter estimated by both the Limited Pareto and Split Linear Rescaling resampling methods. These plots are conventionally used to compare a sample distribution of data to another distribution (normal, lognormal, etc.). The comparison distribution used was the normal distribution, since each of the parameters simulated by the multivariate distribution are normally distributed. The Limited Pareto plots are found in Appendix C, while the Split Linear Rescaling plots are found in Appendix D. By examining the Limited Pareto plots, it can be seen that all six of them had a characteristic shape. Since the series of parameter estimates did not fall on the red line in each plot, it can be understood that the parameter estimates were not representative of a normal distribution. This confirmed the results calculated from the Chi-squared p-value. By examining the Split Linear Rescaling plots, it can be seen that all six of them had a characteristic shape, as well. In these six plots, it is not as easy to conclude that the series of parameter estimates was not representative of a normal distribution; the estimates lie much closer to the red line in each plot. However, the distances between the parameter estimate series and the red lines were sufficiently large enough to reject normality.

<u>Reserve Estimate Analysis</u>

The Excel output from the VaR and TVaR calculations is presented in the ten charts (additively-adjusted and multiplicatively-adjusted estimates for each of the five triangles) in Appendix F. As can be seen from the output, certain patterns can be distinguished. The tail measures of risk from the multivariate normal resampling and Split Linear Rescaling were very similar; the percentages shown in the output did not deviate much from each other. Also, the tail measures of risk from the Limited Pareto resampling generally showed lower percentages. This indicated that the sampling distribution was less extreme in the left tail and had values closer to the projected reserve number. The Limited Pareto tail measures of risk were higher than those from both the multivariate normal and Split Linear Rescaling methods, and the difference between the Limited Pareto and the other two generally widened as higher percentiles were evaluated.

The same output also shows the differences in risk capital needed, and there are relatively consistent patterns discernible from the results. Generally, the multiplicatively-adjusted risk measures are smaller than those from the additively-adjusted method. This would make the multiplicatively-adjusted figures more favorable to use over the additively-adjusted figures. Also, for the majority of the cases, the difference in risk capital between Split Linear Rescaling and the multivariate normal hovers between 1% and 3%, with some instances less than 1% and others greater than 10% and even 20%. Differences between Limited Pareto and the multivariate normal are much more extreme, with some differences as low as 3%, but mostly above 10-20%. The differences escalate as higher tail risk measure percentages are evaluated, with differences spiking to 40-60%.

CONCLUSIONS

As can be seen from the differences in risk capital needed, there is a significant difference in the sampling distributions of the reserve estimates calculated using bootstrapping and those calculated using the asymptotic theory approximation. One of the goals of a P&C insurer is to have high return on investment (ROI), and this ratio can be expressed as *Profit / Risk Capital*. As the risk capital number decreases, ROI increases. For example, if risk capital decreases by 10%, ROI increases by 11.11%. Since many of the ratios are significantly large (especially using Limited Pareto resampling), and due to the fact that even 2% differences (in either direction) in profitability are noteworthy (ratios from approximately 0.98 to 1.02), it can be said that the two methodologies are significantly different in terms of their tail risk measures.

APPENDICES

Appendix A – R Code for Multivariate Normal Distribution

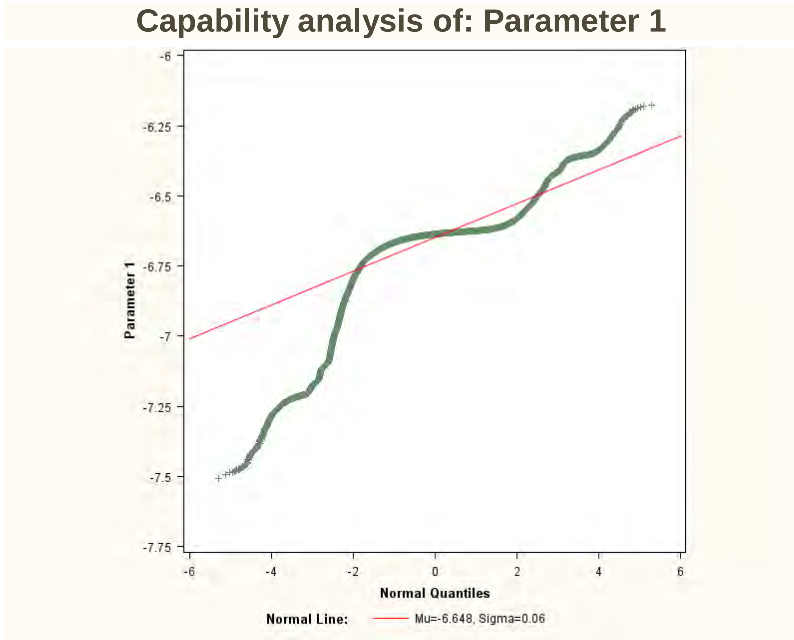
```
library(mvtnorm)
varcov <- read.table("C:/Users/thartl/Documents/Research/Assymptotic Theory Case Study
(DiFronzo)/sigma.txt", sep="\t", header=FALSE)
varcov <- data.matrix(varcov)
colnames(varcov) <- NULL
vct.stdev<-sqrt(diag(varcov))
mu <- read.table("C:/Users/thartl/Documents/Research/Assymptotic Theory Case Study
(DiFronzo)/means.txt", sep="\t", header=FALSE)
mu <- mu[,1]
GetCDF<-function(ind){
return(pmvnorm(mean=mu,sigma=varcov,lower=MkLower(ind),upper=MkUpper(ind)))}
MkLower<-function(ind){
dbl<-ind
d6<-qnorm((dbl %% 10)/10)
dbl<-dbl %/% 10
d5<-qnorm((dbl %% 10)/10)
dbl<-dbl %/% 10
d4<-qnorm((dbl %% 10)/10)
dbl<-dbl %/% 10
d3<-qnorm((dbl %% 10)/10)
dbl<-dbl %/% 10
d2<-qnorm((dbl %% 10)/10)
dbl<-dbl %/% 10
d1<-qnorm((dbl %% 10)/10)
return(vct.stdev*c(d1,d2,d3,d4,d5,d6)+mu)}
MkUpper<-function(ind){
dbl<-ind
d6<-qnorm(((dbl %% 10)+1)/10)
dbl<-dbl %/% 10
d5<-qnorm(((dbl %% 10)+1)/10)
dbl<-dbl %/% 10
d4<-qnorm(((dbl %% 10)+1)/10)
dbl<-dbl %/% 10
d3<-qnorm(((dbl %% 10)+1)/10)
dbl<-dbl %/% 10
d2<-qnorm(((dbl %% 10)+1)/10)
dbl<-dbl %/% 10
d1<-qnorm(((dbl %% 10)+1)/10)
return(vct.stdev*c(d1,d2,d3,d4,d5,d6)+mu)}
lst.copula<-lapply(0:999999,GetCDF)
lst.vals<-sapply(lst.copula, function(m) m[1])
write(lst.vals,"C:/Users/thartl/Documents/Research/Assymptotic Theory Case Study
(DiFronzo)/vals.txt", sep="\n")
```

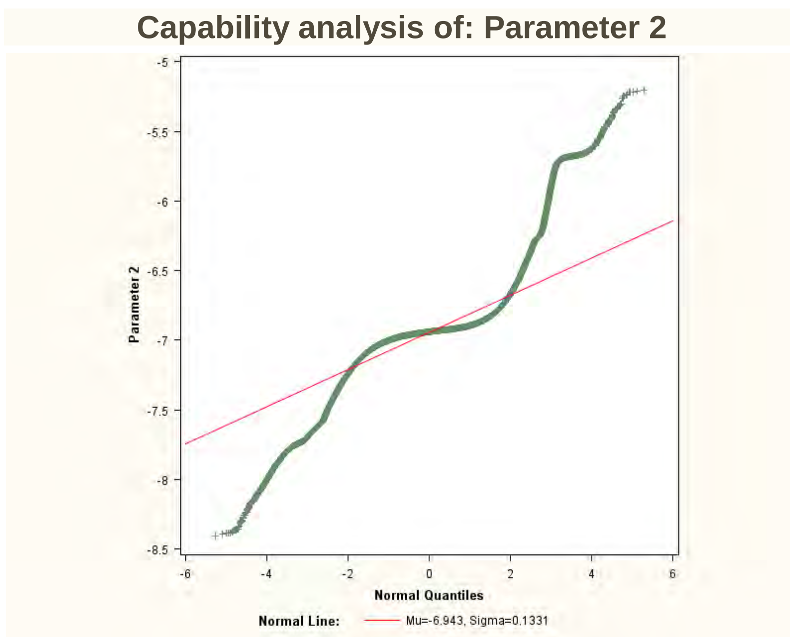
Appendix B – SAS Code for Binning Parameter Estimates

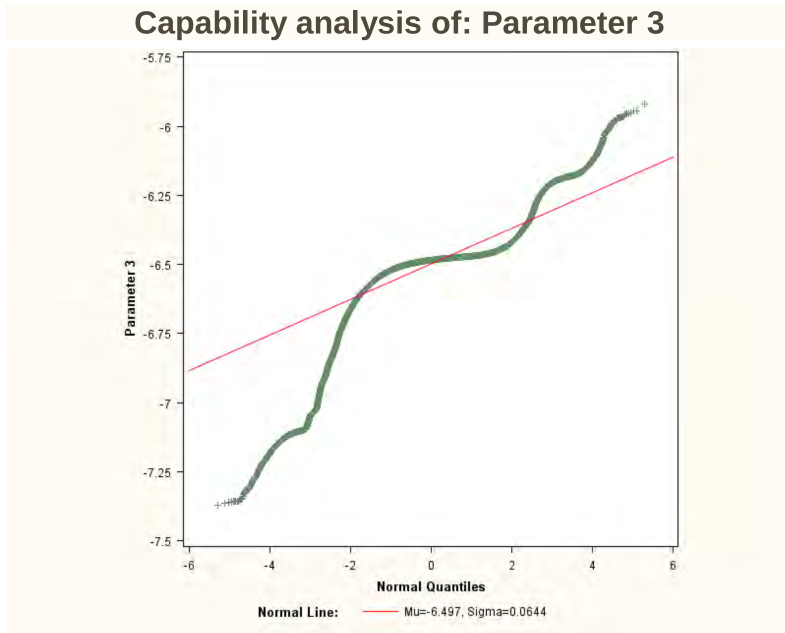
```
options missing='0';

data test1;
infile "C:\Users\student\Documents\ExcelOutput\LPTest(10M).csv"
dlm=",";
input p1 p2 p3 p4 p5 p6 Reserve;
run;

proc rank data=test1 groups=10 out=tested1;
var p1 p2 p3 p4 p5 p6;
ranks rank_p1 rank_p2 rank_p3 rank_p4 rank_p5 rank_p6;
run;

data copula1;
set tested1;
drop p1 p2 p3 p4 p5 p6 Reserve;
identifier=(100000*rank_p1)+(10000*rank_p2)+(1000*rank_p3)+(100*rank
_p4)+(10*rank_p5)+(rank_p6);
run;

proc freq data=copula1;
tables identifier / nocum nopercent out=copula1;
run;

data copula1;
set copula1 (rename=(Count=Count1));
run;

data test2;
infile "C:\Users\student\Documents\ExcelOutput\SLRTest(10M).csv"
dlm=",";
input p1 p2 p3 p4 p5 p6 Reserve;
run;

proc rank data=test2 groups=10 out=tested2;
var p1 p2 p3 p4 p5 p6;
ranks rank_p1 rank_p2 rank_p3 rank_p4 rank_p5 rank_p6;
run;

data copula2;
set tested2;
drop p1 p2 p3 p4 p5 p6 Reserve;
identifier=(100000*rank_p1)+(10000*rank_p2)+(1000*rank_p3)+(100*rank
_p4)+(10*rank_p5)+(rank_p6);
run;

proc freq data=copula2;
tables identifier / nocum nopercent out=copula2;
run;
```

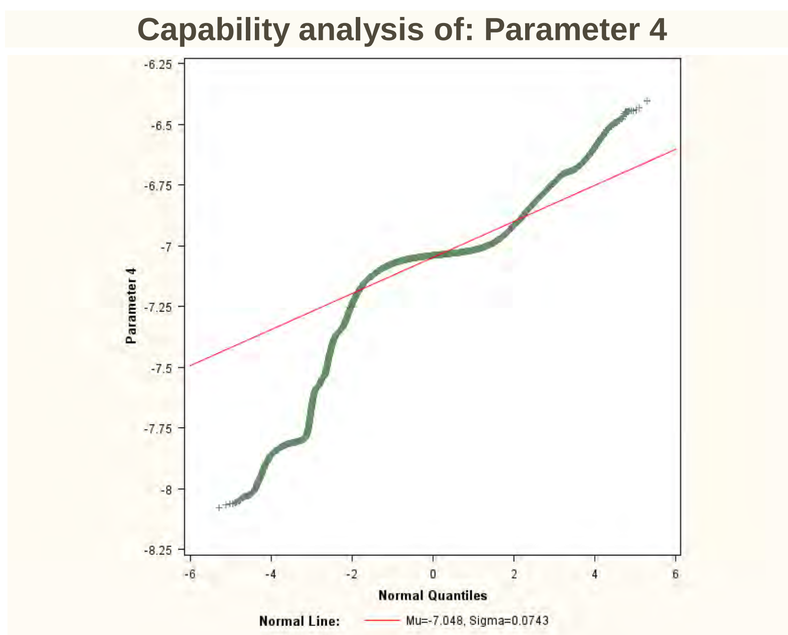
```
data copula2;
set copula2 (rename=(Count=Count2));
run;

data comparison;
merge copula1 copula2;
by identifier;
drop PERCENT;
run;

ods csvall file="C:\Users\student\Documents\Honors
Capstone\ResamplingAnalysis(10)_MergedData.csv";

proc print data=comparison;
run;

ods csvall close;
```

Appendix C – Limited Pareto Q-Q Plots

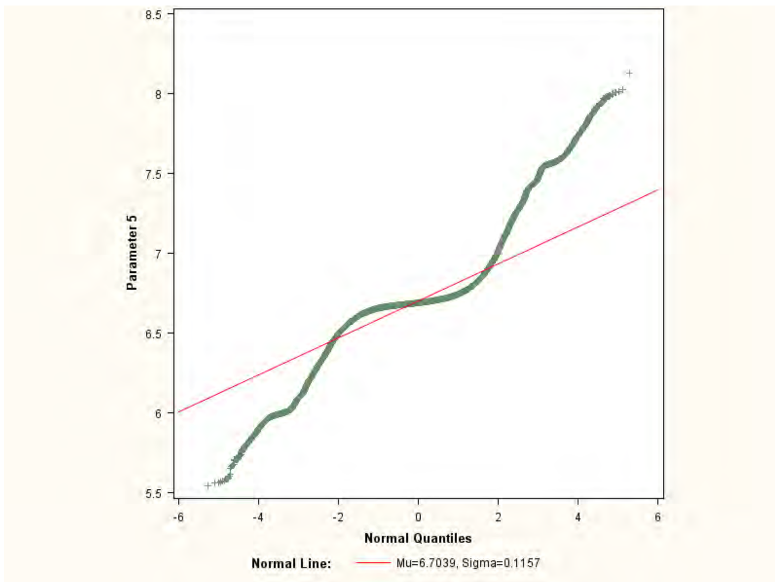




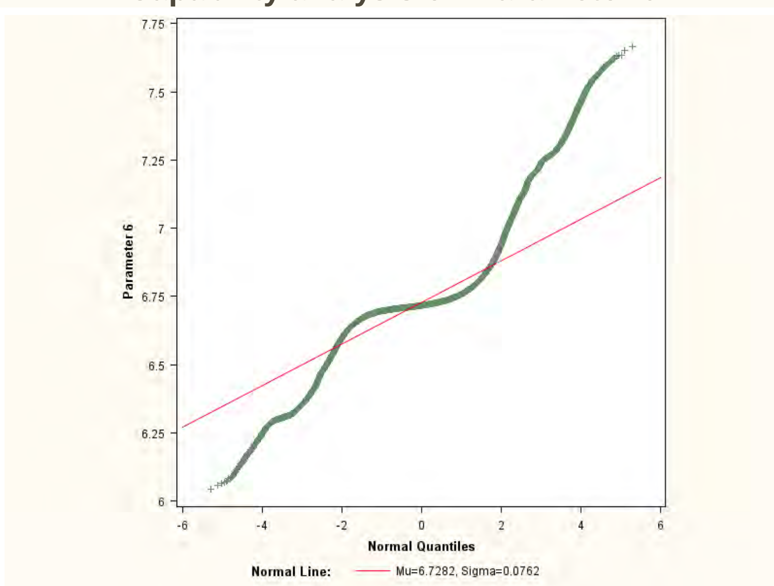

Capability analysis of: Parameter 3
Parameter 3
-5.75
-6
-6.25
-6.5
-6.75
-7
-7.25
-7.5
-6
-4
-2
0
2
4
6
Normal Quantiles
Normal Line:
Mu=-6.497, Sigma=0.0644


Capability analysis of: Parameter 4
Parameter 4
-6.25
-6.5
-6.75
-7
-7.25
-7.5
-7.75
-8
-8.25
-6
-4
-2
0
2
4
6
Normal Quantiles
Normal Line:
Mu=-7.048, Sigma=0.0743

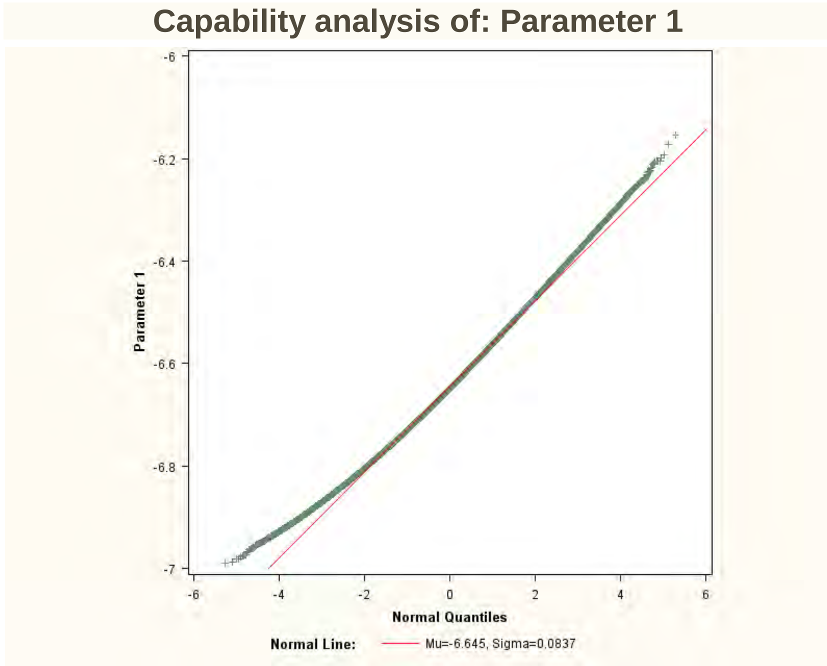
Capability analysis of: Parameter 5

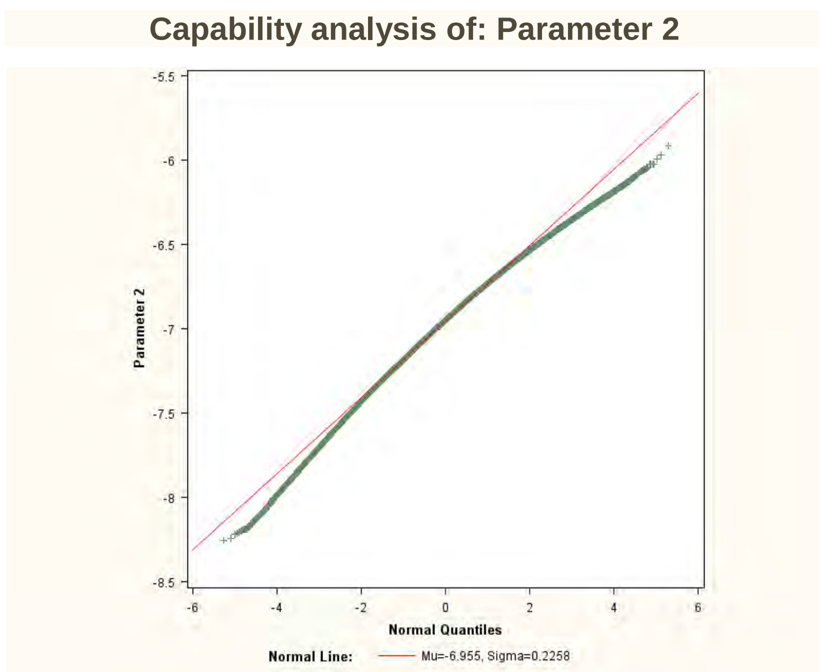


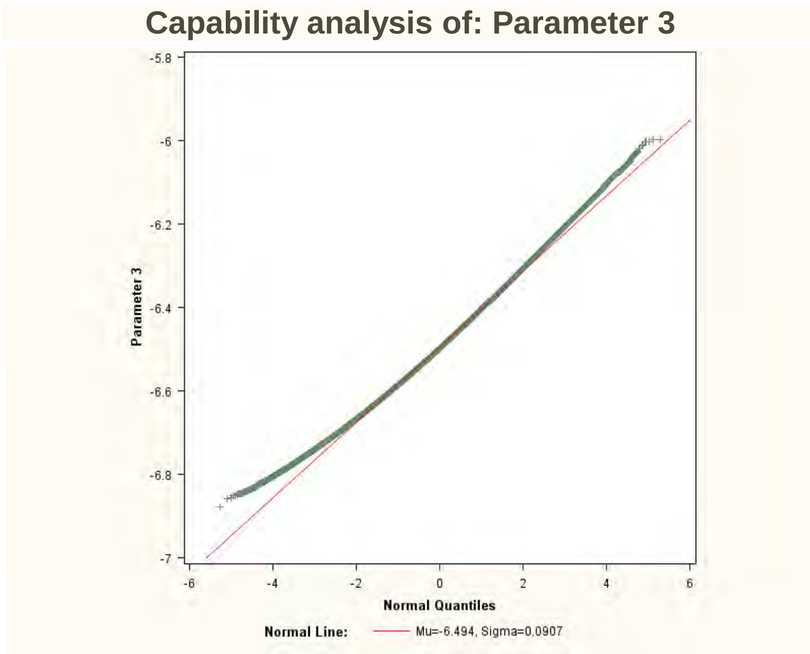
Capability analysis of: Parameter 6

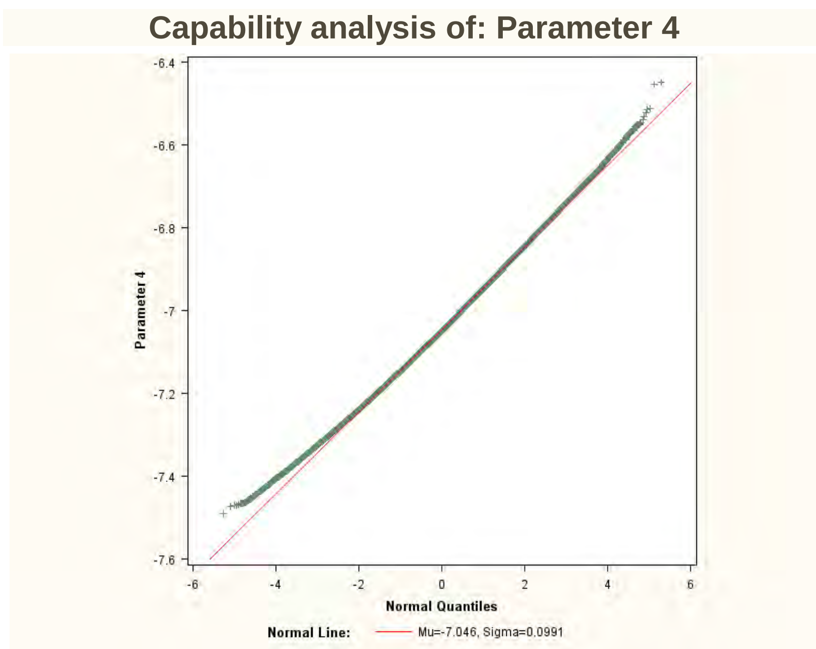


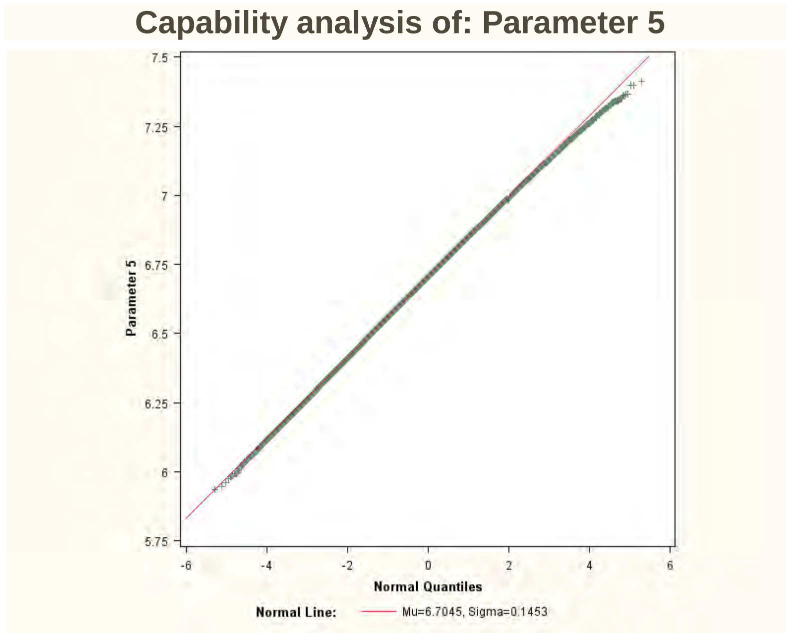
Appendix D – Split Linear Rescaling Q-Q Plots

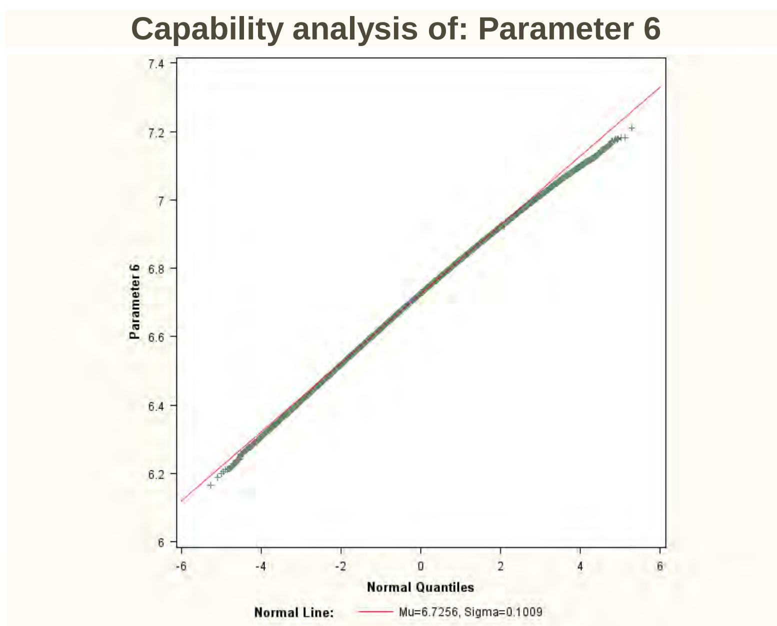





Capability analysis of: Parameter 3
Parameter 3
Normal Quantiles
Normal Line:
Mu=-6.494, Sigma=0.0907


Capability analysis of: Parameter 4
Parameter 4
Normal Quantiles
Normal Line:
Mu=-7.046, Sigma=0.0991


Capability analysis of: Parameter 5
Parameter 5
Normal Quantiles
Normal Line: Mu=6.7045, Sigma=0.1453


Capability analysis of: Parameter 6
Parameter 6
Normal Quantiles
Normal Line: Mu=6.7256, Sigma=0.1009

Appendix E – Triangle Data

Triangle A (incremental)		Taylor & Ashe								
Period	Dev	1	2	3	4	5	6	7	8	9	10
Exp											
1		357,848	719,008	617,974	666,748	467,856	283,583	316,627	150,625	253,245	67,948
2		400,050	842,014	896,226	1,195,112	559,160	483,086	308,404	256,003	399,030	
3		325,082	847,343	1,114,697	991,117	660,957	326,505	363,715	279,899		
4		318,924	1,027,430	901,212	1,142,130	435,503	375,392	387,678			
5		383,148	823,017	1,028,973	745,625	496,104	396,443				
6		344,219	1,053,896	753,391	952,607	587,599					
7		464,785	813,661	1,014,946	1,189,738						
8		377,544	1,153,281	1,333,674							
9		355,772	1,007,522								
10		344,014									

Triangle B (incremental)		Alaska - WC								
Period	Dev	1	2	3	4	5	6	7	8	9	10
Exp											
1		4,608	4,489	2,593	1,718	1,285	620	401	1,235	536	408
2		3,873	4,033	2,197	1,526	847	870	999	964	526	
3		4,488	5,278	2,811	1,928	877	817	488	480		
4		4,302	4,264	2,366	1,446	979	785	485			
5		5,152	5,205	2,336	1,376	681	656				
6		7,496	5,898	3,044	1,602	1,374					
7		7,486	7,351	3,558	1,900						
8		7,401	5,960	3,189							
9		7,772	7,200								
10		6,814									

Triangle C (incremental)		Chubb - PAL								
Period	Dev	1	2	3	4	5	6	7	8	9	10
Exp											
1		69,458	53,502	34,208	20,841	8,630	3,902	1,500	1,642	77	595
2		52,951	45,262	32,176	21,315	11,022	6,370	1,146	792	1,337	
3		46,059	42,425	26,585	17,150	10,056	4,463	2,801	513		
4		42,297	39,254	23,614	14,490	8,403	3,363	1,945			
5		41,479	32,614	26,962	16,208	10,533	2,266				
6		36,376	34,240	20,446	16,444	8,338					
7		37,714	35,011	28,197	15,498						
8		33,457	32,240	22,166							
9		33,172	33,722								
10		37,784									

Triangle D (incremental)		Chubb - CMP									
Period	Dev	1	2	3	4	5	6	7	8	9	10
Exp											
1		241,486	161,157	41,456	44,928	25,187	15,496	7,099	5,292	3,512	3,583
2		382,020	267,774	70,867	41,683	22,497	28,605	4,144	5,969	4,656	
3		256,101	208,198	62,943	33,392	27,522	23,199	11,700	31,442		
4		281,384	190,936	65,389	39,091	25,360	9,877	12,437			
5		452,892	252,514	63,459	48,364	31,437	22,040				
6		257,750	163,351	82,750	48,972	50,567					
7		296,436	181,029	64,858	54,173						
8		431,112	252,447	101,161							
9		283,067	349,872								
10		228,050									

Triangle E (incremental)		ACE 2013 - GL									
Period	Dev	1	2	3	4	5	6	7	8	9	10
Exp											
1		67,641	108,301	98,195	96,212	68,927	77,375	64,443	43,850	22,621	21,495
2		62,463	138,727	128,724	161,519	104,053	237,688	55,113	52,052	44,884	
3		45,902	105,458	140,400	137,795	129,508	109,144	62,639	38,200		
4		46,512	118,497	156,581	268,859	258,671	148,893	105,465			
5		42,217	118,143	187,731	185,476	145,113	190,192				
6		32,855	116,096	143,389	170,335	117,016					
7		47,439	138,701	145,228	127,893						
8		59,858	155,475	136,586							
9		42,038	141,539								
10		50,094									

Appendix F – Tail Measures of Risk Output

Triangle A – Additive Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$19,342,058	0.4	-0.386	-0.415	-0.387	-0.420	-0.354	-0.388	1.002	1.014	0.916	0.936
5	Yes		1.0	-0.347	-0.384	-0.350	-0.388	-0.286	-0.345	1.008	1.010	0.824	0.898
5	Yes		5.0	-0.269	-0.318	-0.268	-0.319	-0.129	-0.214	0.997	1.004	0.479	0.675
5	No	$19,030,487	0.4	-0.256	-0.278	-0.259	-0.284	-0.251	-0.282	1.014	1.021	0.980	1.017
5	No		1.0	-0.231	-0.256	-0.233	-0.260	-0.194	-0.247	1.007	1.016	0.840	0.964
5	No		5.0	-0.172	-0.208	-0.172	-0.209	-0.080	-0.142	0.998	1.006	0.464	0.685
All	Yes	$18,878,244	0.4	-0.363	-0.394	-0.369	-0.401	-0.324	-0.388	1.015	1.017	0.892	0.984
All	Yes		1.0	-0.332	-0.365	-0.333	-0.370	-0.249	-0.323	1.003	1.012	0.749	0.886
All	Yes		5.0	-0.254	-0.301	-0.253	-0.302	-0.128	-0.204	0.995	1.002	0.501	0.676
All	No	$18,680,856	0.4	-0.185	-0.201	-0.190	-0.208	-0.165	-0.189	1.030	1.032	0.893	0.939
All	No		1.0	-0.165	-0.185	-0.169	-0.190	-0.122	-0.161	1.026	1.030	0.737	0.872
All	No		5.0	-0.122	-0.148	-0.124	-0.152	-0.053	-0.094	1.017	1.025	0.434	0.632

Triangle A – Multiplicative Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$19,342,058	0.4	-0.369	-0.397	-0.376	-0.409	-0.348	-0.382	1.019	1.029	0.942	0.961
5	Yes		1.0	-0.333	-0.368	-0.341	-0.377	-0.282	-0.339	1.024	1.025	0.848	0.922
5	Yes		5.0	-0.257	-0.304	-0.261	-0.310	-0.127	-0.211	1.013	1.019	0.492	0.693
5	No	$19,030,487	0.4	-0.247	-0.269	-0.257	-0.280	-0.249	-0.281	1.037	1.044	1.008	1.046
5	No		1.0	-0.223	-0.248	-0.230	-0.257	-0.193	-0.246	1.031	1.039	0.865	0.992
5	No		5.0	-0.166	-0.200	-0.170	-0.206	-0.079	-0.141	1.022	1.029	0.477	0.705
All	Yes	$18,878,244	0.4	-0.350	-0.380	-0.364	-0.395	-0.320	-0.384	1.038	1.040	0.915	1.010
All	Yes		1.0	-0.320	-0.352	-0.328	-0.364	-0.246	-0.320	1.025	1.035	0.768	0.908
All	Yes		5.0	-0.245	-0.290	-0.249	-0.298	-0.126	-0.201	1.018	1.026	0.514	0.693
All	No	$18,680,856	0.4	-0.180	-0.196	-0.189	-0.207	-0.165	-0.189	1.055	1.056	0.916	0.961
All	No		1.0	-0.161	-0.180	-0.169	-0.190	-0.121	-0.161	1.051	1.054	0.755	0.894
All	No		5.0	-0.119	-0.144	-0.123	-0.151	-0.053	-0.093	1.041	1.050	0.445	0.648

Triangle B – Additive Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$ 44,901	0.4	-0.348	-0.376	-0.342	-0.369	-0.315	-0.345	0.985	0.980	0.907	0.918
5	Yes		1.0	-0.316	-0.348	-0.312	-0.343	-0.254	-0.309	0.988	0.984	0.804	0.887
5	Yes		5.0	-0.244	-0.288	-0.239	-0.283	-0.119	-0.192	0.979	0.983	0.489	0.666
5	No	$ 44,569	0.4	-0.226	-0.247	-0.224	-0.247	-0.233	-0.256	0.991	1.001	1.030	1.035
5	No		1.0	-0.203	-0.227	-0.201	-0.226	-0.176	-0.224	0.992	0.996	0.869	0.986
5	No		5.0	-0.151	-0.183	-0.150	-0.182	-0.071	-0.127	0.990	0.994	0.467	0.696
All	Yes	$ 46,255	0.4	-0.315	-0.343	-0.306	-0.333	-0.321	-0.413	0.972	0.972	1.019	1.204
All	Yes		1.0	-0.285	-0.316	-0.278	-0.307	-0.206	-0.315	0.976	0.972	0.724	0.996
All	Yes		5.0	-0.217	-0.258	-0.212	-0.253	-0.108	-0.177	0.980	0.978	0.500	0.686
All	No	$ 54,495	0.4	-0.165	-0.180	-0.167	-0.185	-0.152	-0.192	1.015	1.029	0.926	1.066
All	No		1.0	-0.146	-0.165	-0.148	-0.168	-0.101	-0.149	1.008	1.018	0.693	0.908
All	No		5.0	-0.108	-0.132	-0.107	-0.132	-0.043	-0.079	0.984	1.001	0.396	0.597

Triangle B – Multiplicative Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$ 44,901	0.4	-0.332	-0.360	-0.333	-0.359	-0.309	-0.340	1.002	0.997	0.930	0.944
5	Yes		1.0	-0.302	-0.333	-0.304	-0.333	-0.250	-0.304	1.005	1.001	0.826	0.912
5	Yes		5.0	-0.233	-0.275	-0.232	-0.275	-0.117	-0.188	0.997	1.000	0.502	0.684
5	No	$ 44,569	0.4	-0.220	-0.240	-0.222	-0.245	-0.232	-0.254	1.010	1.020	1.055	1.060
5	No		1.0	-0.197	-0.220	-0.199	-0.224	-0.175	-0.222	1.013	1.015	0.891	1.009
5	No		5.0	-0.146	-0.177	-0.148	-0.180	-0.070	-0.126	1.011	1.013	0.479	0.713
All	Yes	$ 46,255	0.4	-0.305	-0.333	-0.303	-0.330	-0.318	-0.410	0.992	0.992	1.042	1.232
All	Yes		1.0	-0.276	-0.306	-0.275	-0.304	-0.204	-0.312	0.995	0.993	0.739	1.018
All	Yes		5.0	-0.210	-0.250	-0.210	-0.250	-0.107	-0.175	1.001	0.999	0.511	0.701
All	No	$ 54,495	0.4	-0.160	-0.175	-0.167	-0.185	-0.152	-0.192	1.041	1.054	0.949	1.093
All	No		1.0	-0.142	-0.160	-0.147	-0.167	-0.101	-0.149	1.034	1.044	0.711	0.931
All	No		5.0	-0.105	-0.128	-0.106	-0.132	-0.043	-0.078	1.011	1.027	0.407	0.613

Triangle C – Additive Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$206,709	0.4	-0.205	-0.226	-0.210	-0.231	-0.219	-0.276	1.028	1.026	1.068	1.224
5	Yes		1.0	-0.183	-0.206	-0.188	-0.211	-0.139	-0.215	1.025	1.028	0.762	1.046
5	Yes		5.0	-0.134	-0.164	-0.137	-0.168	-0.057	-0.110	1.019	1.024	0.424	0.669
5	No	$211,332	0.4	-0.181	-0.199	-0.179	-0.196	-0.200	-0.250	0.990	0.987	1.109	1.258
5	No		1.0	-0.161	-0.181	-0.161	-0.180	-0.127	-0.192	0.999	0.993	0.790	1.063
5	No		5.0	-0.119	-0.145	-0.117	-0.143	-0.048	-0.096	0.979	0.986	0.401	0.661
All	Yes	$202,167	0.4	-0.195	-0.215	-0.199	-0.219	-0.204	-0.268	1.023	1.022	1.048	1.250
All	Yes		1.0	-0.175	-0.196	-0.177	-0.199	-0.138	-0.203	1.010	1.019	0.790	1.037
All	Yes		5.0	-0.128	-0.156	-0.130	-0.159	-0.052	-0.102	1.016	1.016	0.406	0.654
All	No	$185,236	0.4	-0.146	-0.161	-0.141	-0.155	-0.139	-0.162	0.963	0.962	0.950	1.004
All	No		1.0	-0.130	-0.147	-0.125	-0.141	-0.091	-0.132	0.956	0.961	0.696	0.902
All	No		5.0	-0.096	-0.117	-0.092	-0.112	-0.035	-0.066	0.958	0.959	0.365	0.568

Triangle C – Multiplicative Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$206,709	0.4	-0.202	-0.222	-0.209	-0.230	-0.218	-0.275	1.036	1.034	1.079	1.239
5	Yes		1.0	-0.180	-0.203	-0.186	-0.210	-0.139	-0.214	1.033	1.036	0.771	1.058
5	Yes		5.0	-0.132	-0.161	-0.136	-0.167	-0.057	-0.109	1.029	1.033	0.428	0.677
5	No	$211,332	0.4	-0.175	-0.192	-0.178	-0.195	-0.200	-0.249	1.019	1.014	1.146	1.297
5	No		1.0	-0.156	-0.175	-0.160	-0.179	-0.127	-0.192	1.027	1.021	0.815	1.096
5	No		5.0	-0.115	-0.140	-0.116	-0.142	-0.048	-0.095	1.010	1.015	0.414	0.682
All	Yes	$202,167	0.4	-0.192	-0.212	-0.198	-0.219	-0.204	-0.268	1.032	1.031	1.059	1.263
All	Yes		1.0	-0.173	-0.193	-0.176	-0.199	-0.138	-0.203	1.018	1.028	0.797	1.048
All	Yes		5.0	-0.126	-0.154	-0.130	-0.158	-0.052	-0.102	1.026	1.025	0.409	0.660
All	No	$185,236	0.4	-0.142	-0.156	-0.141	-0.155	-0.139	-0.162	0.991	0.989	0.978	1.033
All	No		1.0	-0.126	-0.142	-0.125	-0.141	-0.091	-0.132	0.985	0.989	0.717	0.927
All	No		5.0	-0.093	-0.113	-0.092	-0.112	-0.035	-0.066	0.988	0.988	0.377	0.585

Triangle D – Additive Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$1,317,995	0.4	-0.508	-0.540	-0.454	-0.485	-0.366	-0.417	0.893	0.898	0.720	0.773
5	Yes		1.0	-0.471	-0.508	-0.418	-0.454	-0.304	-0.368	0.887	0.893	0.646	0.723
5	Yes		5.0	-0.375	-0.433	-0.333	-0.384	-0.166	-0.242	0.887	0.887	0.442	0.561
5	No	$1,194,049	0.4	-0.482	-0.514	-0.401	-0.433	-0.328	-0.370	0.832	0.841	0.681	0.718
5	No		1.0	-0.445	-0.482	-0.365	-0.401	-0.253	-0.320	0.821	0.833	0.570	0.664
5	No		5.0	-0.359	-0.411	-0.283	-0.332	-0.123	-0.198	0.787	0.808	0.344	0.480
All	Yes	$1,320,654	0.4	-0.503	-0.533	-0.447	-0.480	-0.438	-0.545	0.890	0.900	0.871	1.022
All	Yes		1.0	-0.464	-0.502	-0.413	-0.448	-0.296	-0.428	0.890	0.893	0.639	0.851
All	Yes		5.0	-0.369	-0.426	-0.325	-0.378	-0.165	-0.250	0.880	0.887	0.446	0.588
All	No	$ 922,216	0.4	-0.305	-0.328	-0.257	-0.283	-0.198	-0.246	0.841	0.863	0.648	0.749
All	No		1.0	-0.278	-0.306	-0.227	-0.257	-0.136	-0.196	0.818	0.841	0.489	0.640
All	No		5.0	-0.216	-0.254	-0.166	-0.204	-0.069	-0.110	0.768	0.801	0.317	0.432

Triangle D – Multiplicative Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$1,317,995	0.4	-0.440	-0.469	-0.426	-0.455	-0.352	-0.406	0.969	0.971	0.802	0.866
5	Yes		1.0	-0.406	-0.440	-0.390	-0.426	-0.292	-0.355	0.961	0.968	0.718	0.807
5	Yes		5.0	-0.318	-0.371	-0.309	-0.358	-0.158	-0.232	0.973	0.967	0.496	0.627
5	No	$1,194,049	0.4	-0.373	-0.403	-0.381	-0.412	-0.323	-0.364	1.022	1.024	0.865	0.905
5	No		1.0	-0.340	-0.374	-0.346	-0.382	-0.249	-0.315	1.016	1.022	0.732	0.843
5	No		5.0	-0.266	-0.312	-0.266	-0.314	-0.121	-0.194	1.002	1.009	0.455	0.622
All	Yes	$1,320,654	0.4	-0.440	-0.469	-0.435	-0.466	-0.428	-0.534	0.986	0.995	0.972	1.139
All	Yes		1.0	-0.405	-0.440	-0.400	-0.435	-0.289	-0.419	0.989	0.990	0.714	0.951
All	Yes		5.0	-0.318	-0.370	-0.314	-0.366	-0.160	-0.244	0.989	0.990	0.503	0.659
All	No	$ 922,216	0.4	-0.251	-0.272	-0.256	-0.282	-0.197	-0.246	1.017	1.038	0.785	0.903
All	No		1.0	-0.227	-0.251	-0.227	-0.256	-0.136	-0.195	0.999	1.020	0.598	0.777
All	No		5.0	-0.171	-0.205	-0.166	-0.203	-0.069	-0.109	0.968	0.990	0.400	0.534

Triangle E – Additive Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$3,867,195	0.4	-0.492	-0.527	-0.468	-0.498	-0.422	-0.485	0.950	0.946	0.857	0.922
5	Yes		1.0	-0.452	-0.492	-0.429	-0.467	-0.353	-0.424	0.950	0.949	0.782	0.861
5	Yes		5.0	-0.356	-0.415	-0.339	-0.394	-0.170	-0.274	0.953	0.951	0.478	0.660
5	No	$3,877,892	0.4	-0.412	-0.442	-0.409	-0.445	-0.344	-0.397	0.994	1.006	0.836	0.897
5	No		1.0	-0.376	-0.412	-0.372	-0.411	-0.261	-0.337	0.989	0.997	0.693	0.817
5	No		5.0	-0.293	-0.344	-0.283	-0.338	-0.124	-0.204	0.968	0.983	0.425	0.594
All	Yes	$3,454,055	0.4	-0.439	-0.472	-0.411	-0.442	-0.360	-0.408	0.935	0.936	0.820	0.865
All	Yes		1.0	-0.400	-0.439	-0.373	-0.410	-0.301	-0.360	0.933	0.934	0.751	0.820
All	Yes		5.0	-0.310	-0.366	-0.291	-0.342	-0.154	-0.236	0.939	0.934	0.498	0.647
All	No	$3,744,684	0.4	-0.321	-0.345	-0.308	-0.334	-0.260	-0.289	0.961	0.967	0.810	0.836
All	No		1.0	-0.290	-0.320	-0.278	-0.308	-0.205	-0.254	0.957	0.963	0.707	0.794
All	No		5.0	-0.224	-0.264	-0.210	-0.251	-0.092	-0.155	0.939	0.951	0.411	0.585

Triangle E – Multiplicative Bias Adjustment

	Payment	Projected		MVN		SLR		LP		SLR		LP	
Diagonals	Parameter	Reserve	CL%	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR	VaR	TVaR
5	Yes	$3,867,195	0.4	-0.456	-0.489	-0.447	-0.476	-0.411	-0.473	0.979	0.973	0.901	0.968
5	Yes		1.0	-0.419	-0.457	-0.409	-0.446	-0.344	-0.414	0.977	0.977	0.821	0.906
5	Yes		5.0	-0.330	-0.384	-0.323	-0.376	-0.166	-0.267	0.981	0.980	0.503	0.694
5	No	$3,877,892	0.4	-0.373	-0.403	-0.395	-0.430	-0.339	-0.391	1.057	1.067	0.908	0.972
5	No		1.0	-0.341	-0.374	-0.359	-0.396	-0.257	-0.332	1.051	1.059	0.753	0.887
5	No		5.0	-0.264	-0.312	-0.273	-0.326	-0.122	-0.201	1.031	1.045	0.463	0.646
All	Yes	$3,454,055	0.4	-0.415	-0.446	-0.402	-0.432	-0.355	-0.402	0.968	0.970	0.856	0.902
All	Yes		1.0	-0.378	-0.415	-0.365	-0.401	-0.296	-0.355	0.968	0.968	0.784	0.856
All	Yes		5.0	-0.292	-0.345	-0.285	-0.334	-0.152	-0.233	0.973	0.969	0.519	0.675
All	No	$3,744,684	0.4	-0.297	-0.320	-0.304	-0.330	-0.258	-0.286	1.026	1.033	0.870	0.896
All	No		1.0	-0.269	-0.296	-0.275	-0.305	-0.204	-0.252	1.022	1.029	0.759	0.852
All	No		5.0	-0.206	-0.244	-0.208	-0.249	-0.091	-0.154	1.008	1.019	0.444	0.630

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