Bryant University

HONORS THESIS

The Impact of Forces on Knee Ligaments: A Biomechanical Analysis

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ABSTRACT

Anatomy of the human body is complicated, and it impacts human physiology in every way, shape, and form. The application of physics in mechanical models creates simplified systems to help understand more complex structures. This paper looks at how anatomical research informed the creation of a two-dimensional computational model of the knee joint. The existing research and literature on the anterior cruciate ligament (ACL), the posterior cruciate ligament (PCL), the lateral collateral ligament (LCL) and the medial collateral ligament (MCL) is quite plentiful. The posterolateral corner (PLC), however is less studied in both anatomical research and its impact on surrounding structures. Though the field of biomechanics has been gaining more knowledge in recent years about the posterolateral corner of the knee, there are still questions about how it works with the other four ligaments to impact the joint's overall stability. In looking at this question, it was hypothesized that stability of the knee would vary depending on the ligament, or a combination of ligaments, ruptured. It was predicted that in some cases, stability would be possible despite injury to one ligament. In order to test this hypothesis, the model was run under various scenarios, each of which tested the impact on stability when removing one or more ligaments of the knee. The mechanical model showed that humans do not always rely on all four ligaments for a stable knee joint.

INTRODUCTION

Posterolateral corner tears often go undetected in conjunction with anterior cruciate ligament tears (Shahan et al, 1999). Though the field of biomechanics has been gaining more knowledge in recent years about the posterolateral corner of the knee, there are still questions about the connection between PLC tears and ACL tears that have not been answered (Shahan et al, 1999). This dilemma prompts the question: why do posterolateral tears go undetected? Perhaps the better question is: how can those undetected tears be better detected?

Understanding the knee joint's expected stability both before and after certain ligament ruptures can help address the second question. Applying existing anatomical research of knee ligaments to a physics-based approach enables the creation of a computational model that can then be used to predict joint stability under various circumstances. A model like this has the potential to help medical professionals better understand when injuries have occurred despite a seemingly stable joint.

A fair amount of literature exists in which the knee joint is represented through mathematical or computer models. Early literature shows that it is possible to utilize two-dimensional models to emulate the knee joint and the effects that external forces have on said joint (Moeinzadeh, Engin, Akkas, 1983). A little over fifteen years after the two-dimensional model was proven effective, engineers began focusing on three-dimensional models to help describe the biomechanics of the knee joint (Piazza & Delp, 2001).

Additionally, many studies surrounding the biomechanics of the knee joint as a whole have been done. In one study it was shown that ligaments could appropriately be modeled as elastic springs while the contact forces could be represented in the same direction as the normal force (Moeinzadeh, Engin & Akkas, 1983). Another study echoed this modeling technique of ligaments by representing the collateral and patellar ligaments as elastic tensile springs (Piazza & Delp, 2001).

Using Hooke's Law to model ligaments as elastic tensile springs, while valid, requires accurate spring constants (Urone, 2012). Hooke's law reads:

$$F = k * \Delta L \quad (1)$$

In this equation, the force, F, is a function of the ligament's spring constant, k, multiplied by the change in ligament length, L. The change in ligament length is quantified as the change from the ligament's original length at zero degrees flexion to the existing length at the angle of bending (Urone, 2012). In looking at the ACL, one group of researchers found the ligament's average length to be 27.5 ± 2.8 mm in people 48-86 years old and 26.9 ± 1.5 mm in people 16-26 years old (Noyes & Grood, 1976). They also determined that the spring constant for the ACL was 129 ± 39 N/mm in the older age group and 182 ± 56 N/mm in the younger age group (Noyes & Grood, 1976). Another group of researchers did a similar study years later and found that the spring constant for the ACL was 242 ± 28 N/mm (Woo et al, 1991). They found that the maximum load the ACL could take on before ultimate failure was 2160 ± 157 N (Woo et al, 1991). The extreme variability in these studies shows that anatomy is complex and far from standardized.

This trend continues when looking at the other three major ligaments of the knee joint. For the PCL, one study found the spring constant to be 117.29 ± 8.90 N/mm (Forsyth et al, 2016). That same group of researchers discerned the maximum load was 1974.88 ± 752.73 N (Forsyth et al, 2016). Similar studies were done for the MCL and LCL. The spring constant for the MCL in one study was 63 ± 14 N/mm and 59 ± 12 N/mm for the LCL (Wilson et al, 2012). The maximum loads for the MCL and LCL respectively were documented as 799 ± 209 N and 392 ± 104 N (Wilson et al, 2012). These researchers also noted the extreme variability of anatomical structures, citing nine other studies that all found different spring constants and ultimate loads for the collateral ligaments (Wilson et al, 2012). In one final study, a researcher modeling the knee joint used 100 N/mm for the ACL and PCL spring constant and 65 N/mm for the MCL and ACL (Ozada, 2015). All of these examples show how great the variability in anatomical structures can be.

Other studies have been found to use nonlinear force equations to determine loads placed on ligaments. The nonlinear force equation is:

$$F = k * \Delta L^2 \quad (2)$$

The force, *F*, exerted on each ligament is a function of the spring constant, *k*, multiplied by the change in length, *L*, squared (Moeinzadeh, Engin & Akkas, 1983). The difference between Hooke's law and the nonlinear equation is the squaring that happens to the change in length in the latter equation. This function changes the spring constants that can be used in each equation. Those used in Hooke's Law will have units of N/mm while those used in the nonlinear equation will have units of N/mm^2. In one study the spring constants for this second equation were found to be 15 N/mm^2 for the LCL and MCL, 30 N/mm^2 for the ACL, and 35 N/mm^2 for the PCL (Moeinzadeh, Engin & Akkas, 1983). This shows that there are multiple ways to mathematically model the ligaments.

The length of ligaments also varied substantially throughout the literature. Many researchers who were cited for their spring constant values also had ligament lengths or coordinate systems in place. One coordinate system in particular looked at the ligaments in a three-dimensional plane (Edwards, Lafferty & Lange, 1970).

Going back to some of the ultimate loads cited earlier, it becomes apparent that the ligaments of the knee joint take on various loads when external forces are applied (Moeinzadeh, Engin & Akkas, 1983). The anterior cruciate ligament (ACL) takes on a higher load than the collateral ligaments do (Moeinzadeh, Engin & Akkas, 1983). Additionally, the function of the ACL proved to be to resist forward displacement of the tibia (Moeinzadeh, Engin & Akkas, 1983). The same study noted that the posterior cruciate ligament prevents backward displacement of the tibia while the collateral ligaments were there to resist varus and valgus rotation (Moeinzadeh, Engin & Akkas, 1983).

Each of these findings enabled the creation of an accurate anatomical system by applying the proper physics calculations and biomechanical properties of the knee joint to the models at hand. Understanding the purpose of specific elements of the knee joint alongside the physical properties was crucial in accurately depicting the knee joint.

MATERIALS AND METHODS

Drawing the 2D two-ligament model force diagrams

Pulling from the literature, we started by drawing a simple, two-ligament force diagram of the left knee including only the cruciate ligaments. This model looked like an "x" in its simplest form as seen in Figure 1.

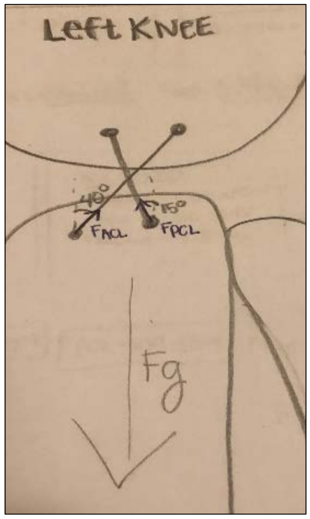


Fig 1. Simple system sketched out of the cruciate ligaments.

Pulling the drawing into a coordinate system, we came up with the model seen in Figure 2 that was then used to plug into the computer model.

Building the 2D two-ligament computer model

The coordinate system was then translated into a program using Python that was used to model the system in motion. The program, referenced in Appendix 2.1, was built so that the spring constant for each ligament could be changed alongside the attachment sites of each ligament. The model would then assess the total force exerted on each ligament throughout a side-to-side bending motion, something that is somewhat unnatural for the knee, but nevertheless tests the systems from a physics point of view. The equation written to test the two ligaments are as follows with equation (3) representing the force of the PCL and equation (4) representing the force on the ACL.

```
F_CB=spring_force(B,C,Lo)     (3)
F_EA=spring_force(A,E,Lo)     (4)
```

These equations use the coordinates from Figure 2 as inputs to determine the ligament's length as the angle of bending changes.

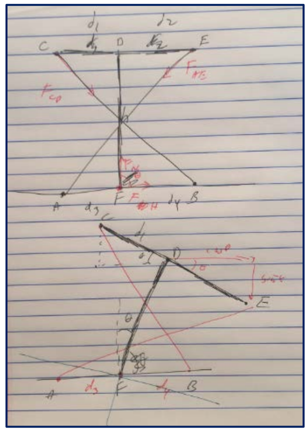


Fig 2. Simple diagram of the ACL and PCL sketched out into a coordinate system.

The program also calculates net torque exerted on the system as a whole to show the overall stability of the joint.

Numerous anatomical factors were left out of consideration at this point with the understanding that they would be worked into the model later on. Some of these factors included the correct spring constants and attachment sites for the left knee.

<u>Drawing the 2D four-ligament model force diagrams</u>

Eventually, we added the collateral ligaments to the initial system. We started off drawing force diagrams on paper again, as seen in Figure 3, to better understand how the forces played together with one another.

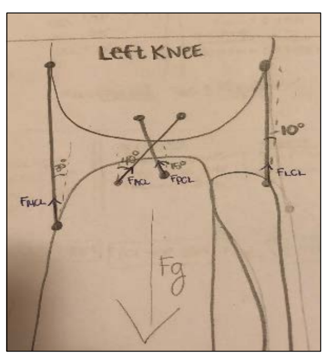


Fig 3. Sketch of four-ligament system.

The four-ligament model was then converted into a simpler coordinate system, much like the one built for the two-ligament model, and then translated into the computer model.

Builing the 2D four-ligament computer model

Taking the two-ligament Python notebook, seen in Appendix 2.1, the collateral ligaments were added into the coordinate system and their subsequent force equations were built into the new notebook, as seen in Appendix 2.2. The following two equations show the force equation for the LCL the MCL respectively, as documented in the program.

F_IG=spring_force(G,I,Lo) (5)

F_JH=spring_force(H,J,Lo) (6)

The coordinate points seen in these equations are pulled from the coordinate system, seen in Figure 4.

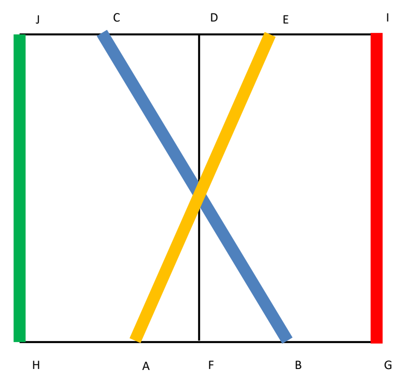


Fig 4. Four-ligament coordinate system built into Python model.

The spring constants and specific lengths of the model were refined only after the model was functioning.

Creating the 3D four-ligament model

The final step in this process was creating a three-dimensional model to help ascertain the stability of the knee joint in flexion and extension in addition to the lateral bending that was already assessed in the two-dimensional model. The notebook for this model can be found in Appendix 2.3. The primary difference between this model and the others was the altercation of the coordinate system which then affected the force equations for each ligament. Figure 5 shows the ligaments in the new grid as represented in the Python program. Here, the coordinate system cited in earlier literature was used as the third plane was built in to represent the knee joint (Edwards, Lafferty & Lange, 1970).

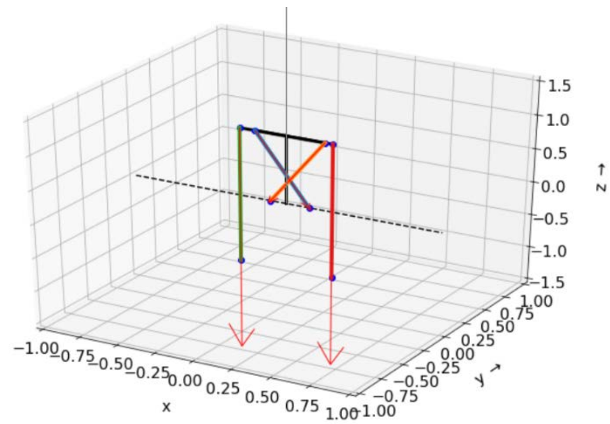


Fig 5. Three-dimensional, four-ligament coordinate system built into Python model.

This model was built to assess the feasibility of using more accurate physics-based systems in future studies to see what happens to the net torque during both lateral bending and flexion-extension bending.

RESULTS

Testing the stability of the two-ligament model

Before running the computer model, we used the program *Algodoo*, to create a reference point for expected stability. This can be seen in Figure 6. We noted the structure falling over when it was left sitting by itself, indicating that the two cruciate ligaments would not be enough to maintain full knee stability.

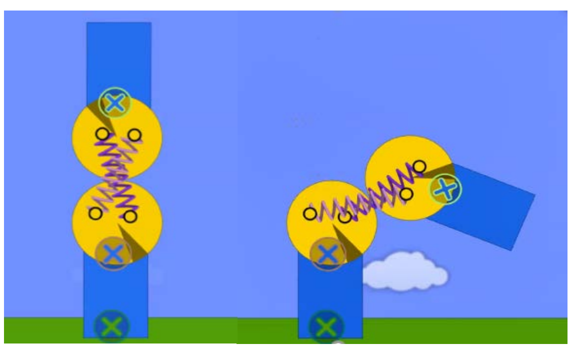


Fig 6. Simple, two-ligament knee system in an Algodoo model both standing tall (left) and falling over (right) to show instability.

We then ran the program we built which resulted in the plot shown in Figure 7. This plot shows the net torque on the knee joint as a function of lateral bending.

When the knee bends from left-to-right, the angle at which it is bent is recorded along the x-axis. The ability for the system to stabilize itself is indicated by the trajectory of the net torque. When the net torque is negative, indicated by a red line, the knee is moving in a clockwise motion, or toward the left side of the body. When the net torque is positive, indicated by a blue line, the knee moving in a counterclockwise motion, or toward the right side of the body. In a stable knee, one would expect to see the net torque working in the opposite direction as lateral bending. As the knee bends toward the right side of the body, from -40º to 0º along the x-axis, the expected net torque would be clockwise, or toward the left to counteract the bending angle and stabilize the knee. The same is true for the opposite scenario. As the knee bends toward the left side of the body, from 0º to 20º, the expected net torque would be in the counterclockwise direction to stabilize the system.

In looking at the plot for the two-ligament joint in Figure 7, the net torque continues to become more and more negative. This indicates that the knee is not able to stabilize during lateral bending, especially when the angle of bending becomes positive to indicate bending toward the left. The knee under these conditions would be unstable.

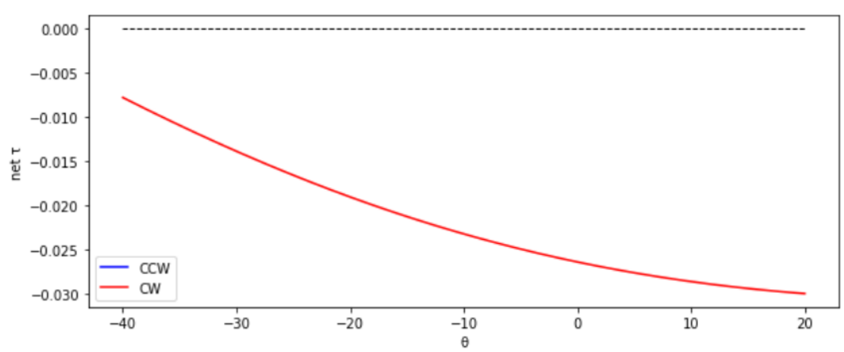


Fig 7. Net torque of the knee as a function of lateral bending with only the ACL and PCL present.

<u>Testing the stability of the four-ligament model</u>
The plot in Figure 8 shows the net torque of the knee with all four ligaments as a function of the same lateral bending that was seen in the two-ligament model. Again, when the torque is negative, represented by a red line, the knee is working to pull itself in the clockwise or

leftward direction. When the torque is positive, indicated by a blue line, the knee system is working to pull itself toward the right or counterclockwise. Figure 8 shows that when all four ligaments are present and the knee is bending toward the right side of the body, from -40º to 0º, the torque is stabilizing the system by pulling the knee back toward the left. As the knee starts bending toward the left, from 0º to 20º, the knee can again stabilize itself by pulling the system toward the right. This indicates that the four-ligament knee system is stable.

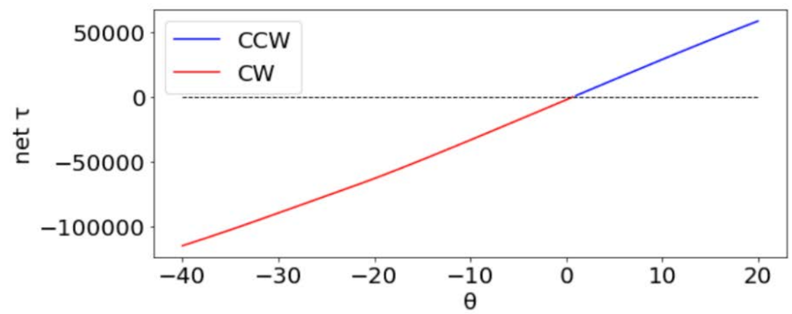
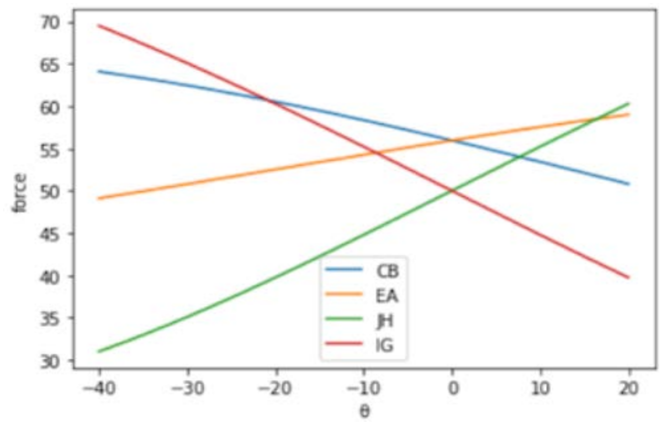


Fig 8. Net torque of knee as a function of lateral bending with all four ligaments present.

Testing the function of the four-ligament model

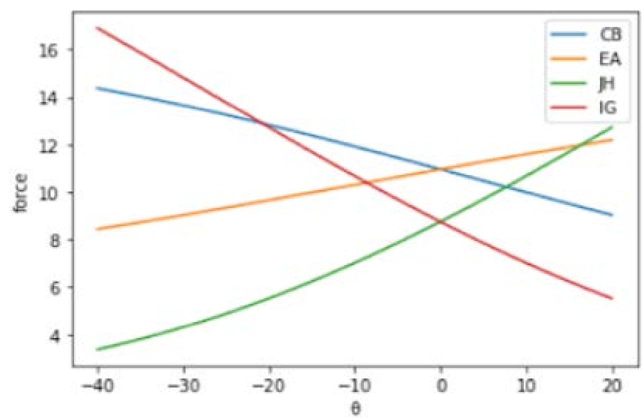
Once the model was representative of the basic physics going on at the knee joint, more detailed human anatomy was built into it. The literature shows that both linear and nonlinear force equations can be used to determine the force exerted on each ligament. We tested the two equations side-by-side. For Hooke's law, we used 100 N/mm to represent the spring constant of all four ligaments, and for the non-linear equation we used 65 N/mm^2 to represent the ligament's spring constants.

Figure 9 shows the force exerted by each ligament as a function of lateral bending of the left knee when the ligament is modeled using Hooke's Law. Figure 10 shows the same graph, this time with the ligaments modeled as non-linear springs. The extreme variability in the two shows that either one or both the equations were not accurate representations of the joint. Hooke's Law yielded forces roughly four times greater than the non-linear equation did.



Hooke's Law

Fig 9. Force exerted on each ligament as a function of Hooke's law for each angle of lateral bending.



Non-Linear Spring

Fig 10. Force exerted on each ligament as a function of the nonlinear spring equation for each angle of lateral bending.

The resulting forces appeared to be calculated with ligament that were less than 1 unit in length. Still, the discrepancy between the two was quite large for comparing the same spring. Reevaluating a series of spring constants at anatomically accurate lengths we realized that the numbers varied over too large of a range to get consistent readings between these two equations. We chose to move forward with Hooke's Law for the force equation as it is more consistently used in other literature and was less sensitive to large changes in ligament length.

In addition to noting the discrepancy between the two formulas, we recognized that having one spring constant to represent all four ligaments was not accurate, so we went into the program and distinguished between two spring constants, one for the cruciate ligaments and one for the collateral ligaments. The new equations for the PCL, ACL, LCL, and MCL read respectively as follows:

F_CB=spring_force(B,C,Lo,k1) (7)

F_EA=spring_force(A,E,Lo,k1) (8)

F_IG=spring_force(G,I,Lo,k2) (9)

F_JH=spring_force(H,J,Lo,k2) (10)

Finally, we noted the lack of realistic forces exerted on the ligaments. The literature cites maximum loads on the PCL, ACL, LCL, and MCL as 1974.88 ± 752.73, 2160 ± 157, 799 ± 209, and 392 ± 104 respectively (Forsyth et al, 2016; Woo et al, 1991; Wilson et al, 2012). We therefore adjusted the initial coordinate system to represent realistic ligament lengths. Up until this point, the coordinate system had been used to represent the basic mechanics of the system without inputting accurate ligament attachment sites. Using data from Edwards, Lafferty and Lange's study in 1970, we change the ligament coordinates. Figure 11 reflects the force exerted on each ligament as a function of the changing angle with the new ligament attachment coordinates in the program.

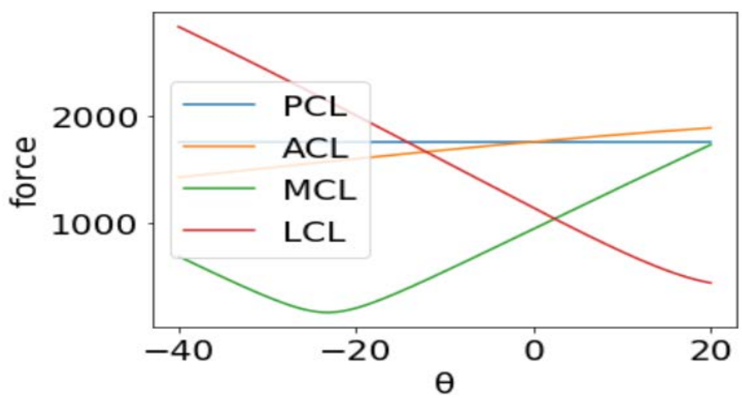


Fig 11. Force exerted on each ligament modeled as a function of Hooke's law and with the correct attachment sites at each angle of lateral bending.

Comparing this force diagram to the ultimate loads cited, it became clear that ligaments can only bear lateral bending from roughly 0º to 20º. Even then, the MCL and LCL cannot always tolerate the force exerted on them at all the angles within this range. The model still serves as a tool to test stability throughout the entire range of motion.

Testing knee stability in accordance with the removal of one ligament

We started testing the joint's stability when various ligaments were removed. In order to simulate the removal of a ligament, the force equation was zero-ed out by changing the formula as seen in Figure 12.

```
F_CB=spring_force(B,C,Lo,k1*1e-40)  # <==
F_EA=spring_force(A,E,Lo,k1)  # <==
F_JH=spring_force(H,J,Lo,k2)  # <==
F_IG=spring_force(G,I,Lo,k2)  # <==
```

Fig 12. Line 1 shows an equation zero-ed out in Python while the other three lines show the normal force equations.

We started by looking at the effects of removing one individual ligament while leaving the other three intact. Once the results were gathered, we began taking out pairs of ligaments to see how that would affect the overall stability. Then, ran models where all but one ligament was removed from the joint. All combination of ligament tears amongst the four major ligaments were assessed.

After verifying a stable system, the model was then able to run as ligaments were removed. The model was run first without the PCL to assess knee stability which can be seen in Figure 13. Then, the program was run without the ACL. Figure 14 shows the net torque on the left knee with only the PCL, LCL, and MCL. When these two simulations were plotted, the net torque showed a stable system, despite missing one ligament.

Conversely, when only the LCL was removed, as shown in figure 15, or only the MCL was removed, as shown in figure 16, stability was not maintained throughout lateral bending. Because we are looking at the lateral bending, the MCL and LCL appear to have a more significant impact on stability than the ACL and PCL do. Figure 15 shows that when the LCL is removed, the knee is not able to pull itself back up when bending from the midline as the net torque goes in the counterclockwise direction from 30º all the way through 0º. This means that if the knee was to bend in that direction, it would fall over. In contrast, the knee would be able to pull itself back to the midline if bending to the left were to occur. The opposite trend appears when the MCL is removed which makes sense given the two ligaments work on opposite sides of the knee joint. Here, Figure 16 shows that the knee is able to pull itself back up from rightward bending as net torque works in the clockwise direction. However, as soon as the knee shifts to bend the other direction, net torque continues to be clockwise, meaning that the knee has lost stability at that point.

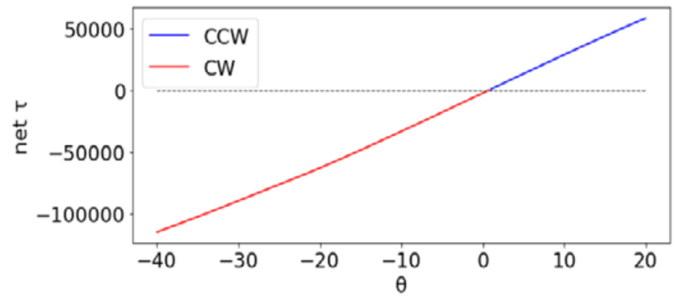


Fig 13. Net torque of knee with all ligaments present except the PCL as a function of lateral bending.

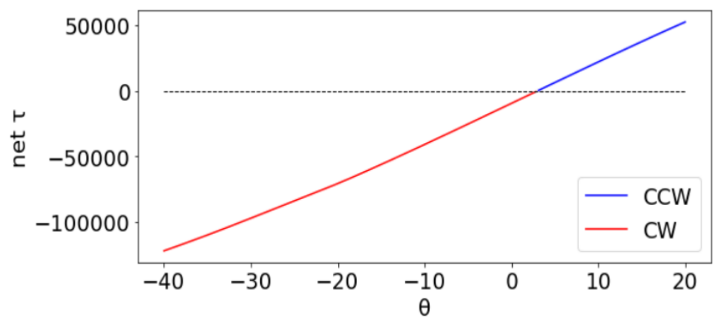


Fig 14. Net torque of knee with all ligaments present except the ACL as a function of lateral bending.

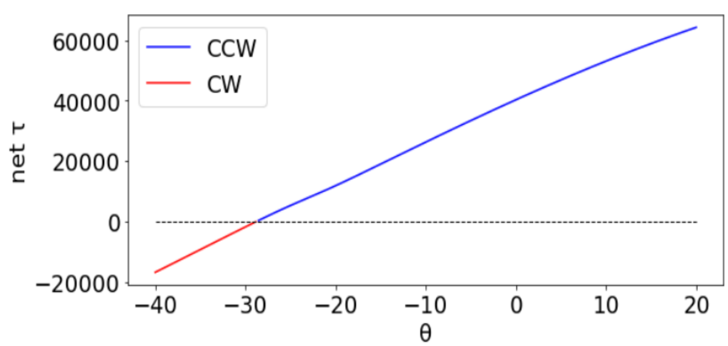


Fig 15. Net torque of knee with all ligaments present except the LCL as a function of lateral bending.

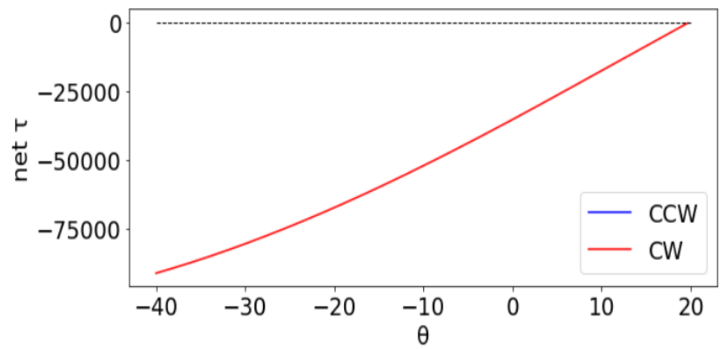


Fig 16. Net torque of knee with all ligaments present except the MCL as a function of lateral bending.

Altogether, these models show that we do not always need all four knee ligaments for the joint to be stable. The model indicates that stability will be maintained in lateral bending in the absence of the ACL or the PCL. Stability will not be maintained completely, however, if the LCL or MCL are removed.

<u>Testing knee stability in accordance with the removal of two ligaments</u>
Once the trials were run for single ligament removal, the program was reset, and pairs of ligaments were removed. This was done to once again to see if stability could be maintained in the absence of two ligaments. Knowing that collateral ligaments have a more profound impact on stability in lateral bending, it makes sense that when these ligaments were removed alongside one other ligament, stability would in some way be impacted. Taking away pairs of ligaments yielded more instability than removing just one ligament which is to be expected.

In terms of removing one cruciate ligament with one collateral ligament, the stability of the net torque plot most often reflected that of the plot in which only said collateral ligament was removed. For example, Figure 17 in which the PCL and LCL were removed, and Figure 18 in which the ACL and LCL were removed, both reflect the plot of just the LCL being removed. This means that the knee only really has stability in bending toward the left. The same trend occurs in Figure 19 and Figure 20 in which the MCL was removed alongside the ACL and PCL respectively. Both of these net torque plots appear similar to the plot in which the MCL was removed alone, with instability occurring as the knee bends to the right.

The only scenario in which stability was maintained in lateral bending was the removal of both the ACL and the PCL as seen in Figure 21. It is important to note that function of the ACL and PCL have more to do with stabilization in flexion and extension which means that the knee may not be completely stable without them. This is something that can be further explored in the three-dimensional model.

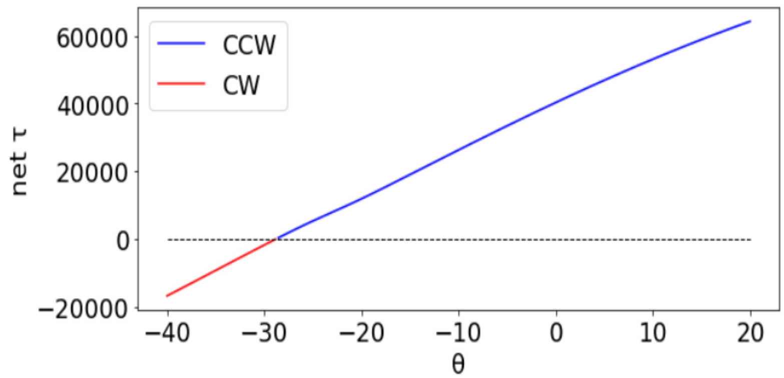


Fig 17. Net torque of knee when the PCL and LCL are removed as a function of lateral bending.

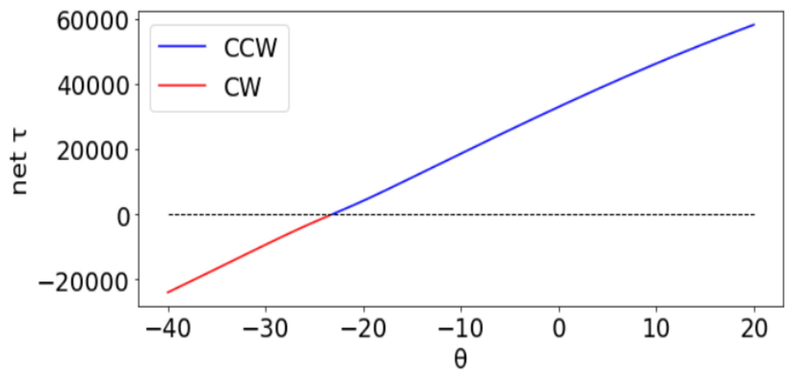


Fig 18. Net torque of knee when the ACL and LCL are removed as a function of lateral bending.

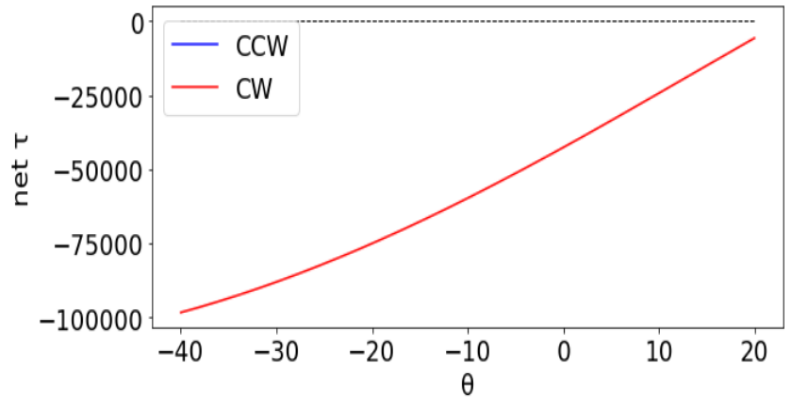


Fig 19. Net torque of knee when the ACL and MCL are removed as a function of lateral bending.

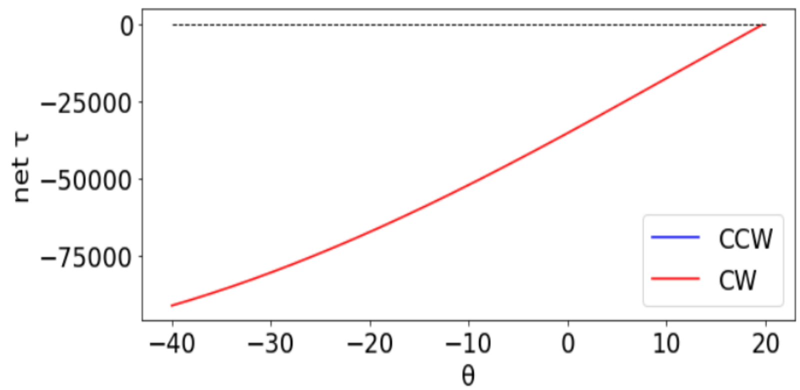


Fig 20. Net torque of knee when the PCL and MCL are removed as a function of lateral bending.

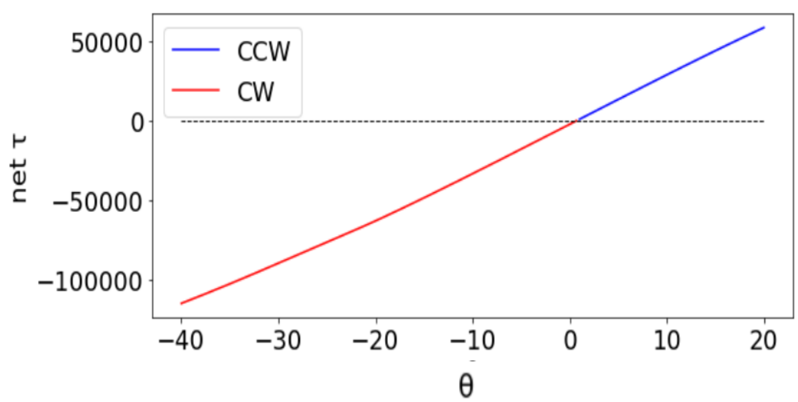


Fig 21. Net torque of knee when the PCL and ACL are removed as a function of lateral bending.

Altogether, removing two ligaments at a time show that the knee has a harder time maintaining stability when more than one ligament is removed. The collateral ligaments appear to have a larger impact on stability than the cruciate ligaments in lateral bending.

<u>Testing knee stability in accordance with the removal of three ligaments</u>

After running the model in the absence of two ligaments, three ligaments were removed to test the knee's stability. Knowing that there was hardly any situation in which the knee had maintained stability without two ligaments, it makes sense why removing three ligaments resulted in minimal stability as well. Figure 22 and Figure 23 show what happens when only the PCL and ACL remain in the knee respectively. Both plots show that there is no stability in the knee joint whatsoever as the net torque is not able to counteract the movement of the knee joint across various angles of bend. When only the LCL remained intact, the plot mirrored that of the plot in which only the MCL was removed as seen in Figure 25 and vis versa when only the MCL remained intact as seen in Figure 24.

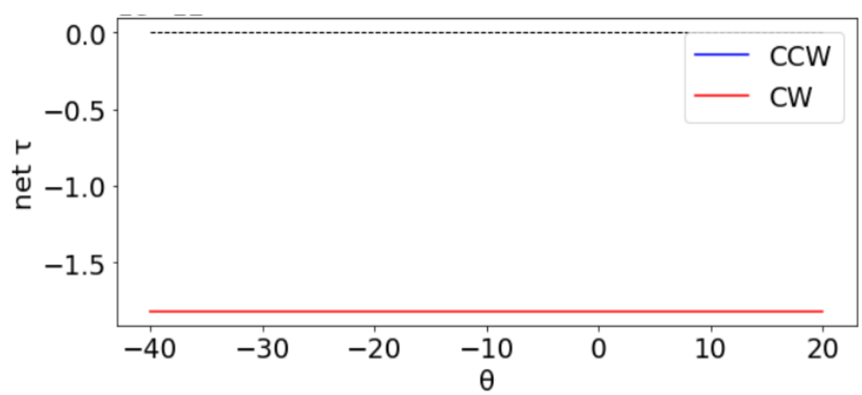


Fig 22. Net torque of knee with only the PCL as a function of lateral bending.

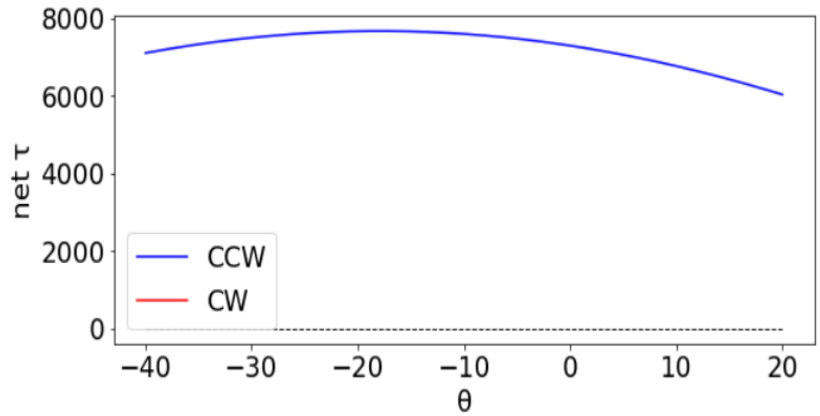


Fig 23. Net torque of knee with only the ACL as a function of lateral bending.

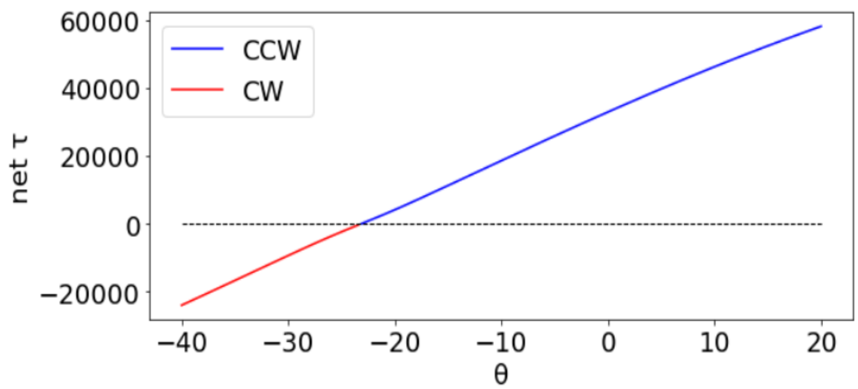


Fig 24. Net torque of knee with only the MCL as a function of lateral bending.

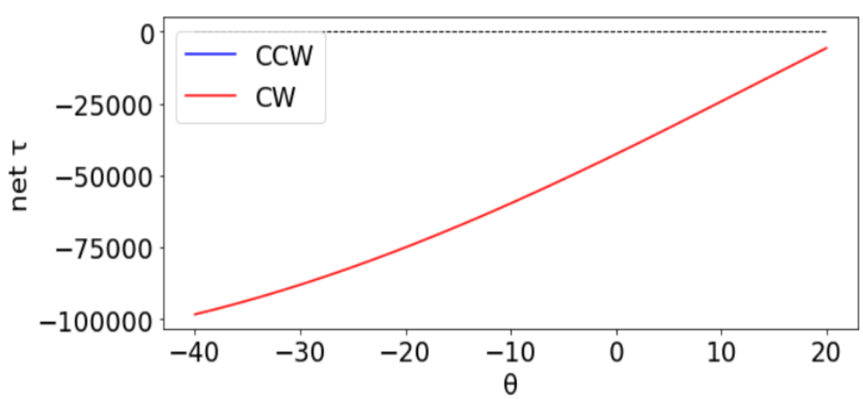


Fig 25. Net torque of knee with only the LCL as a function of lateral bending.

Altogether, we see that the knee joint cannot maintain stability when three ligaments are removed.

DISCUSSION

A computational model of the knee joint shows that we do not always need all four ligaments to be stable. Removing more than one ligament at a time, however, will cause instability in some way. Taking away either the posterior cruciate ligament or the anterior cruciate ligament will enable one to maintain stability in lateral bending. While this model only looked at one plane of movement, it can be expanded to testing in the three-dimensional plane to look at flexion and extension stability.

All that being said, building a very simplistic computational model of the knee joint still proved quite trying. There was extreme anatomical variation that must be considered when looking at the results. While the parameters of ligament attachment sites and spring constants can easily be changed within the model, only one set of measurements was used from a very broad range of referenced points. Likewise, this model, in all its complexities, only accounted for the mechanics of the four major ligaments. It purposefully left out the impacting forces from all other muscles, tendons, and bones in the knee joint. However, these structures still exist within human knees and will impact stability in some way. One such structure, the posterolateral corner, proved too complex to put into the model during this study. In the future, with more anatomical research done about what is and is not included in this structure, it can be built into the model to assess some of the remaining questions about instability alongside anterior cruciate ligament tears.

Despite some of these insufficiencies, this model still proves useful in better understanding where to expect stability despite injuries occurring and where to expect instability. This study also shows just how complicated it is to standardize anatomy. Even then, attempting to turn anatomical systems into simple physics-based models can yield some overarching insights.

APPENDICES

Appendix A—Review of the literature

Review of literature on modeling

A fair amount of literature exists in which the knee joint is represented through mathematical or computer models. Within those models, there are multiple methodologies that take place. Early literature shows that it is possible to utilize two-dimensional models to emulate the knee joint and the effects that external forces have on said joint (Moeinzadeh, Engin, Akkas, 1983). The purpose of this study was to utilize mathematics to map the x-rays of a knee to see whether or not it was possible to determine biodynamic functions of the knee from a two-dimensional model of the joint (Moeinzadeh, Engin, Akkas, 1983). From this study, it was determined that the biodynamic model is useful in identifying the purpose of major knee ligaments and the contact forces that exist within the knee joint (Moeinzadeh, Engin, Akkas, 1983).

In looking more in-depth at the two-dimensional models, a four-bar cruciate linkage model was developed to better understand the relationship between the anterior cruciate ligament and the posterior cruciate ligament (Kraeutler et al, 2018). Building off of this model, a group of researchers put together a 2-D diagram depicting both the ACL versus PCL relationship in addition to the MCL versus LCL relationships (Chittajallu & Kohrt, 1996). This was created mathematically to depict the force relationship between the two pairs of ligaments. Another version of the four-bar cruciate ligament model was established in 2016, this time taking into account the effect of bones and cartilage on the force relationship between the ACL and PCL (Dathe et al, 2016).

A little over fifteen years after the two-dimensional model was proven effective, engineers began focusing on three-dimensional models to help describe the biomechanics of the knee joint (Piazza & Delp, 2001). One study that undertook this task also looked to increase the complexity of knee models by including both the cartilage and menisci into models (Piazza & Delp, 2001). Through this study, a model was successfully created using two software packages to map and track the motion of ligaments and muscles while emulating a step-up motion (Piazza & Delp, 2001). The model was able to successfully embody the human motion, becoming the first three-dimensional model to capture dynamic knee activity throughout the entirety of the knee joint (Piazza & Delp, 2001). As time has gone on, three-dimensional modeling has become the preferred way of collecting data because they provide more information (Madeti, Chalamalasetti & Bolla Pragada, 2015).

As time went on, studies began to integrate more complex anatomical behaviors into models to further refine the accuracy of data and conclusions. In 2011, a study was done to improve an analytical model which was originally built under the assumption that the center of gravity is held constant during the squatting motion (Fekete et al, 2011). Taking an already existing model, the group of researchers were able to alter the analytical model to simulate movement that was quantitatively closer to experimental data points of human movement than the first model showed (Fekete et al, 2011). Likewise, another study took an existing model of the human knee and looked specifically at the kinematics associated with cruciate ligaments to better emulate their physiological properties (Amiri et al, 2010). The study found that while most models were using a two-bundle representation of the cruciate ligaments, a more detailed, multiple-bundle model significantly altered the way ligaments were portrayed during flexion (Amiri et al, 2010). That being said, the study proposed the new model as a better way to emulate the knee during loading and dynamic motions (Amiri et al, 2010).

From this literature, it is clear that computational modeling has been tested and refined to provide an accurate way in which the knee can be mapped and biomechanically understood. Engineers and scientists alike have continued to work on this technology so that it is a reliable source from which data can be empirically acquired to test the mechanics of the knee under various circumstances. That being said, the literature supports the use of biomechanical models as a means for this project.

<u>Review of literature on posterolateral corner and other ligaments</u>

In looking at the biomechanics of the posterolateral corner of the knee, a great deal of information was acquired to better understand what research exists and what is left to be discovered.

According to multiple research studies done in the past two decades, the anatomical and physiological understanding of the posterolateral corner of the knee has been poorly understood (Shahan et al, 1999; Kang et al, 2018; & Schinhan et al, 2011). This is in part accredited to the misclassification of the PLC in textbooks that went on for a number of years (LaPrade, 2006). As a result, many medical professionals unknowingly overlooked the structure in research (LaPrade, 2006). In 1999, the structure resurfaced in textbooks. That same year, the popliteofibular ligament's existence was proven as part of the PLC, and its role was defined as a major static stabilizer of the knee (Shahan et al, 1999). Using cadaver knees alongside a technique of releasing and reconstructing subsequent ligaments, it was found that the popliteofibular ligament helps to prevent the knee from turning inward and sliding backward (Shahan et al, 1999). However, it was noted that the ligament's reconstruction was not enough to stabilize the knee alone as the lateral cruciate ligament was the primary restraint against the inward motion and was therefore necessary to establish knee stability overall (Shahan et al, 1999). A study done in 2017 echoed these findings, showing that the lateral cruciate ligament was the primary restriction against internal rotation (Dominick et al).

Another study that came out around the same time as the first study mentioned looked at an additional element of the posterolateral corner of the knee, the popliteus muscle. The study looked at the effect of the muscle in two regards: how the muscle's forces impact the posterior cruciate ligament and how the muscles forces impact the kinematics of the knee (Harner et al, 1998). When the popliteus muscle was not present in the knee and the posterior cruciate ligament was put under the stress of a load, the ligament was exposed to significantly more stress than when that same load was applied in the presence of both the ligament and the muscle (Harner et al, 1998). This proved that the popliteus muscle plays an important physiological role in helping the posterior cruciate ligament stop the tibia from sliding backward (Harner et al, 1998). That being said, the study found that the popliteus muscle served as a secondary restraint, being able to stabilize the knee only somewhat when the posterior cruciate ligament was ruptured (Harner et al, 1998).

Going off of that, in a case study done a few years later, an isolated rupture of the popliteus muscle did not inhibit the stability of the knee when it came to internal and external rotation

(Guha, Gorgees & Walker, 2003). After confirming the tear with arthroscopic surgery, the patient underwent a rehabilitation program which enabled him to have full stability and range of motion one year later (Guha, Gorgees & Walker, 2003). This further articulates that the popliteus muscle provides secondary stabilization of the knee as the knee joint is functional without the muscle present (Guha, Gorgees & Walker, 2003). In looking further at the popliteus muscle, another study found that the muscle varies in size and shape for different populations which may further impact the biomechanical and physiological properties of the muscle (Buitrago, Quintero & Ernesto, 2018).

After a few more years of research, a study in 2011 pointed to the presence of a third main structures associated with the posterolateral corner: the fibular collateral ligament (Schinhan et al, 2011). This acts as a static stabilizer alongside the popliteofibular ligament (Schinhan et al, 2011). At the same time, the study pointed to the popliteus muscle as the popliteus muscle-tendon unit (PMTU) and declared it a dynamic stabilizer in the PLC (Schinhan et al, 2011). The study points to the PMTU as the element of the knee that adjusts balance during double and single leg standing at certain angles of flexion (Schinhan et al, 2011). These results indicate that the PMTU is essential for stabilization during the bending portion of a squat or motions like downhill walking (Schinhan et al, 2011). A study done in 2018 further articulated the importance of the popliteus tendon, citing that it was found to be injured in up to 68% of people with posterolateral instability (Kang et al, 2018). The study found that the popliteus tendon was a primary stabilizer of the knee during external rotation and backward sliding (Kang et al, 2018).

In 2017 a study was done to further understand the stabilizing role of all PLC elements (Domnick et al, 2017). The study paid particular attention to what happens when the PLC tears in conjunction with other ligaments like the lateral cruciate ligament and the posterior cruciate ligament (Domnick et al, 2017). The study cited another element of the PLC, the arcuate complex (Domnick et al, 2017). This complex was proven to help secure the popliteofibular ligament, a static stabilizer, when external rotational loads are applied (Domnick et al, 2017). That being said, the arcuate complex and the popliteofibular ligament are noted as the structures that undergo damage when the PLC is deemed ruptured (Domnick et al, 2017). These findings coincide with a suggestion made by the previously mentioned study from 2018 suggesting that the popliteofibular ligament may act as part of the popliteus tendon complex rather than being an independent structure (Kang et al, 2018). The arcuate complex was also shown to be a secondary stabilizer for the back of the knee when the posterior cruciate ligament is torn (Domnick et al, 2017). As previously mentioned, it was also proven that the PLC does not have a primary role in restricting internal rotation when the lateral cruciate ligament is present (Domnick et al, 2017).

Altogether, these studies provide a comprehensive understanding of the posterolateral corner of the knee and its subsequent elements. The information provides a physiological map of the posterolateral corner and enables us to understand the mechanisms of each individual part.

Review of literature on ACL tears in conjunction with PLC tears

Another element of the literature that has yet to be discussed is the allusion to posterolateral corner tears in conjunction with anterior cruciate ligament tears. One article stated that an anterior cruciate reconstruction is destined for failure when the posterolateral corner is torn or injured in some regard (Shahan et al, 1999). This same article mentioned that the cause of this failure was unknown however (Shahan et al, 1999).

Additional articles noted that there are currently tests established to assess damage to the PLC, however they are rarely used during diagnosis as the ACL tear overshadows the injury, leading to an isolated ACL reconstruction (Chahla et al, 2016). This article again echoed that because of the structure of the knee joint, failing to address a PLC tear in an anterior cruciate ligament reconstruction almost always leads to the re-tearing of the ACL (Chahla et al, 2016). Altogether, this information emphasizes the importance of better understanding how to detect injuries that are often masked.

Review of literature on biomechanics of the knee joint

In addition to understanding the mechanics of the posterolateral corner of the knee, a comprehensive study of literature surrounding biomechanics of the knee joint as a whole was undertaken. Here, more focus was directed toward the four major ligaments of the knee joint: the anterior cruciate ligament, the posterior cruciate ligament, the lateral collateral ligament and the medial collateral ligament.

After compiling the literature, it became apparent that there are three main forces at play in the knee joint: contact forces, externally applied forces, and ligament forces (Moeinzadeh, Engin & Akkas, 1983). In one study, it was apparent that ligaments could appropriately be modeled as elastic springs while the contact forces could be represented in the same direction as the normal force (Moeinzadeh, Engin & Akkas, 1983). This study found that as the knee's flexion angle increases, the femoral contact forces does too (Moeinzadeh, Engin & Akkas, 1983). In contrast, the tibial contact point had varying slopes during the flexion motion (Moeinzadeh, Engin & Akkas, 1983). Another study echoed this modeling technique of ligaments by representing the collateral and patellar ligaments as elastic tensile springs (Piazza & Delp, 2001). In this same study however, the contact forces were modeled as three-dimensional meshes that took on the shape of triangles (Piazza & Delp, 2001). These polyhedral meshes were modeled after the contact points in knee implants (Piazza & Delp, 2001).

Looking at ligaments as elastic springs more closely, it becomes evident that Hooke's law is an effective model to use in determining the force exerted on said structure (Urone, 2012). In order to apply Hooke's law, one group of researchers gathered information on the ACL's structural properties (Noyes & Grood, 1976). They found an average length of the ligament to be 27.5 ± 2.8 mm in people 48-86 years old and 26.9 ± 1.5 mm in people 16-26 years old (Noyes & Grood, 1976). They also determined that the spring constant for the ACL was 129 ± 39 N/mm in the older age group and 182 ± 56 N/mm in the younger age group (Noyes & Grood, 1976). Another group of researchers did a similar study years later and found that the spring constant for the ACL was 242 ± 28 N/mm (Woo et al, 1991). This same study found that the max load the ACL could take on before ultimate failure was 2160 ± 157 N (Woo et al, 1991). The extreme variability in these studies shows that anatomy can be complicated and is far from standardized.

This trend continued when looking at the other three major ligaments of the knee joint. In looking at the posterior cruciate ligament in reference to its elastic spring constant, one study found the value to be 117.29 ± 8.90 N/mm (Forsyth et al, 2016). That same group of researchers discerned the maximum load was anywhere within a range of 1974.88 ± 752.73 N

(Forsyth et al, 2016). Similar studies were done for the medial and lateral collateral ligaments. The spring constant for the MCL in one study was 63 ± 14 N/mm and 59 ± 12 N/mm for the LCL (Wilson et al, 2012). The maximum load for the MCL and LCL respectively were documented as 799 ± 209 N and 392 ± 104 N (Wilson et al, 2012). The group of researchers who investigated the ligament properties of the collateral ligaments also noted the extreme variability of anatomical structures within their paper, citing a chart of nine other studies that all found varying spring constants and ultimate loads for the collateral ligaments (Wilson et al, 2012). In one final study, a researcher modeling the knee joint used 100 N/mm for the ACL and PCL spring constant and 65 N/mm for the MCL and ACL (Ozada, 2015). All of these examples show just how great the variability in anatomical structures can be.

In addition to elastic spring constants, other studies have been found to use nonlinear spring constants and their respective force equations to determine loads placed on ligaments. In one study done by a group of researchers, the spring constants were found to be 15 N/mm^2 for the LCL and MCL, 30 N/mm^2 for the ACL, and 35 N/mm^2 for the PCL (Moeinzadeh, Engin & Akkas, 1983). This shows that there are multiple ways to mathematically model the ligaments.

In similar fashion to spring constants, the length of ligaments varies substantially throughout the literature. Many researchers who were cited for their literature on spring constant values also had ligament lengths or coordinate systems in place. One coordinate system in particular looked at the ligaments in a three-dimensional plane (Edwards, Lafferty & Lange, 1970).

Turning now to the second force on exerted on the knee, contact forces, further research has identified ground reaction forces as the main contact force (Soutas-Little, 2008). These forces were further determined to be made up of three component forces, vertical forces, anterior or posterior forces, and medial or lateral forces (Soutas-Little, 2008). In addition to that, an analysis of these contact forces on the knee isolated when specific component forces exert the most influence on the joint during gait. The study found that vertical ground reaction forces are exerted during flexion and anterior ground reaction forces are exerted during extension (Soutas-Little, 2008). On another note, researchers were also able to isolate the force of friction placed on knee ligaments which furthered the understanding of contact forces within the knee (Steinbrück et al, 2014).

In terms of the external forces that are naturally applied to knee joints, one study found that there are four main loads (Fekete et al, 2011). The study found that the quadricep forces, the tibiofemoral forces, the patellofemoral joint reaction forces, and the patellar ligament forces all have significant impacts on the knee (Fekete et al, 2011). That same study found it acceptable to negate frictional and static forces during movement alongside forces of inertia (Fekete et al, 2011). Another study determined the compressive forces could also be negated (Moeinzadeh, Engin & Akkas, 1983). Other studies have corroborated this research, showing the variety of external loads that are placed on the knee joint across a variety of motions (Nisell).

Going back to some of the ultimate loads cited earlier, it became apparent that the ligaments of the knee joint take on various loads when external forces are applied (Moeinzadeh, Engin & Akkas, 1983). The anterior cruciate ligament (ACL) takes on a higher load than the collateral ligaments do (Moeinzadeh, Engin & Akkas, 1983). Additionally, the function of the ACL proved to be to resist forward displacement of the tibia (Moeinzadeh, Engin & Akkas, 1983). The same study noted that the posterior cruciate ligament was intended on preventing backward displacement of the tibia while the collateral ligaments were there to resist varus and valgus rotation (Moeinzadeh, Engin & Akkas, 1983).

Each of these findings enables the creation of an accurate anatomical system by applying the proper physics calculations and biomechanical properties of the knee joint to the models at hand. Understanding the purpose of specific elements of the knee joint alongside the physical properties is crucial in accurately depicting the knee joint.

Appendix B—Python Notebooks

S1 Data.

See attached notebook for the 2D two-ligament model.

S2 Data.

See attached notebook for the 2D four-ligament model.

S3 Data.

See attached notebook for the 3D four-ligament model.

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