1976

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J. Biomechanics, 1976, Vol. 9, pp. 339–342. Pergamon Press. Printed in Great Britain

ON FREQUENCY-DEPENDENT DAMPING COEFFICIENTS IN LUMPED-PARAMETER MODELS OF HUMAN BEINGS*

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Abstract—A three-degree-of-freedom model of the human body, which included both vertebral and visceral paths between the seat and head, was formulated and programmed for solution using the IBM Continuous System Modeling Program. The parameters representing spring stiffnesses and damping coefficients were varied until the body-to-seat or head-to-seat acceleration ratios converged to existing experimental data at corresponding input frequencies. The model exhibited linear behavior for input frequencies up to approx. 6 Hz and a possible transition region for input frequencies between 6 and 10 Hz. For input frequencies between 10 and 30 Hz, visceral damping was found to vary parabolically with input frequency.

INTRODUCTION

In a lumped-parameter model (Fig. 1) for the response of seated humans to sinusoidal seat displacements (Muksian and Nash, 1974) the inclusion of cubic stiffness and damping gave reasonable agreement with the experimental data published by Goldman and Von Gierke (1960) and Pradko *et al.* (1966, 1967). The agreement was good for input frequencies between 1 and approx. 7 Hz and was diverging between approx. 7 and 30 Hz. In order to gauge the sensitivity of the model to damping coefficients, they were arbitrarily increased by 50% of the originally estimated values. The results showed that a good fit to the experimental data would be obtained by using the original damping coefficients for input frequencies up to approx. 8 Hz and the higher damping coefficients for input frequencies greater than 8 Hz. One implication of these results is that frequency-dependent damping coefficients should be included in lumped-parameter modeling of humans. This implication is strengthened by the fact that muscle forces are dependent upon the frequency of stimulation until a tetanus reaction results (Ruch and Fulton, 1960).

This paper presents the results of an investigation to expand the model given by Goldman (1960), which is shown in Fig. 2, and was an investigation preliminary to the model of Muksian (1970). The model for this investigation is shown in Fig. 3, and a comparison with Fig. 2 shows that the basic difference is the

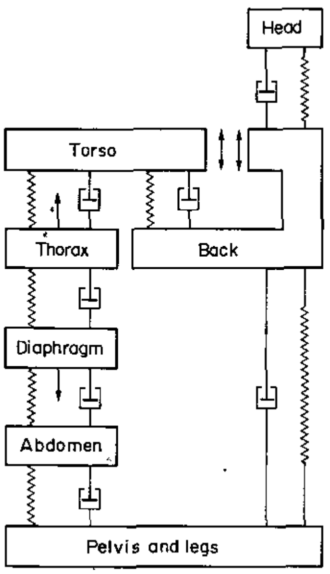

Fig. 1. A nonlinear model of the human body in the sitting position. Muksian and Nash (1974).

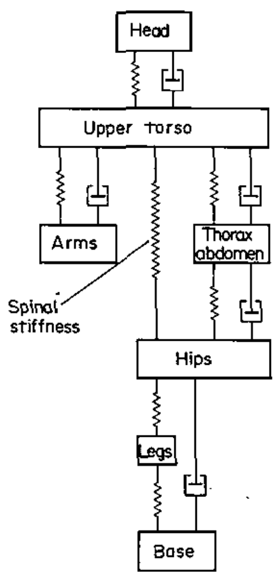

Fig. 2. A mechanical analogue of the human body (after Goldman and Von Gierke, 1960).

* *Received* 27 *January* 1975.

assumption of parallel paths (vertebral and visceral) between the pelvis and head.

EQUATIONS OF MOTION

From free-body-diagrams of the rigid bodies of Fig. 3. Newton's second law gives equations of motion in the form

$$m_i z_i'' = \sum F_i(t) - \sum_j D_{ji} - \sum_j S_{ji} \qquad i = 1,2,3, \quad (1)$$

where j is summed over the number of elements attached to the ith mass.

Spring and viscous damping forces, due to relative displacements and velocities, respectively, between coupled bodies, were assumed to be cubic in nature and are given by

$$S_{ji} = \sum_p K_{pj}(z_i - z_n)^p \qquad p = 1,3 \quad (2)$$

and

$$D_{ji} = \sum_p C_{pj}(z_i' - z_n')^p \qquad p = 1,3, \quad (3)$$

where the subscript n represents the mass coupled to the ith mass by the jth spring or dashpot.

Substitution of equations (2 and 3) into equation (1) gives the equations of motion as

$$m_i z_i'' = \sum F_i(t) - \sum_j \sum_p C_{pj}(z_i' - z_n')^p - \sum_j \sum_p K_{pj} \times (z_i - z_n)^p \qquad i = 1,2,3. \quad (4)$$

In this investigation, the input was a sinusoidal displacement of the seat given by

$$z_3 = z_{3m} \sin(2\pi f_3 t), \quad (5)$$

where z_{3m} is the maximum input displacement, and it is given by

$$z_{3m} = \frac{Gg}{(2\pi f_3)^2}, \quad (6)$$

which allows the maximum input acceleration to be expressed in terms of body weight, G's. Then, in equations (4)

$$z_3 = \frac{Gg}{(2\pi f_3)^2} \sin(2\pi f_3 t), \quad (7)$$

$$z_3' = \frac{Gg}{2\pi f_3} \cos(2\pi f_3 t), \quad (8)$$

and equations (4) are reduced by one since the motion of the pelvis is imposed. In addition, there are no external forces $F_i(t)$. Thus, the acceleration of the ith mass is given by

$$z_i'' = -\frac{1}{m_i}\left\{\sum_j \sum_p C_{pj}(z_i' - z_n')^p + \sum_j \sum_p K_{pj}(z_i - z_n)^p\right\} \qquad i = 1,2. \quad (9)$$

PARAMETER DETERMINATION

All solutions to the equations of motion (9) were obtained using the IBM System 360 version of the Continuous System Modeling Program (CSMP) which is a digital computer simulation of an analog computer.

The maximum input acceleration was held constant at 0.5 g giving the input displacement as

$$z_3 = \frac{0.5g}{(2\pi f_3)^2} \sin(2\pi f_3 t) \quad (10)$$

and the input velocity as

$$z_3' = \frac{0.5g}{2\pi f_3} \cos(2\pi f_3 t). \quad (11)$$

The algorithm for the CSMP solution is

$$z_i'' = -\sum_j \sum_p \frac{C_{pj}}{m_i}(z_i' - z_n')^p - \sum_j \sum_p \frac{K_{pj}}{m_i}(z_i - z_n)^p \qquad i = 1,2 \quad (12)$$

$$z_i' = \int_0^t z_i''\, dt, \quad (13)$$

$$z_i = \int_0^t z_i\, dt, \quad (14)$$

and

$$AR = z_i''/z_3'', \quad (15)$$

where equations (15) represent peak-to-peak acceleration ratios relative to the seat (pelvis) at steady-state. The masses of the head, body, and pelvis were determined from Hertzberg and Clauser (1964) as 5.44 kg (12 lb) for the head, 47.17 kg (104 lb) for the body, and 27.22 kg (60 lb) for the pelvis (which includes the legs) based on a total mass of 79.83 kg (176 lb).

The procedure for estimating the parameters for spring stiffnesses and damping coefficients was as follows. Based on subjective responses to whole-body vibrations given by Magid *et al.* (1962), it was assumed that, during vibrations, the primary resistance is physiological (muscular reactions), and in a static situation, the response is primarily mechanical (deflection of the vertebral column). Magid *et al.* (1962) show a "resonant" subjective response occurring between 6 and 7 Hz input frequencies. Accordingly, in a classic two-mass system, K_{p3} of Fig. 3 would be the sum of vertebral and visceral stiffnesses, and assuming an undamped natural frequency of 6.6 Hz (the average of 6 and 7 Hz appeared to be too ideal), K_{13} is computed as 90476 N/m. To determine vertebral stiffness, the investigator, Muksian, measured his evening (just before retiring) height and morning (immediately upon arising) height for several days and determined his static deflection to be approx. 19 mm. Based on an upper torso weight of 516 N, this gave a vertebral stiffness of 27158 N/m, which, when subtracted from the total stiffness, gives

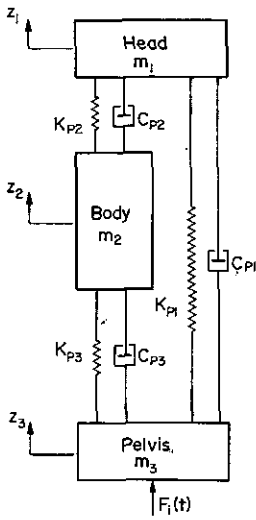

Fig. 3. A dual pelvis to head path model of the human body in the sitting position.

a body stiffness of 63318 N/m. In order to estimate the damping coefficients, the linear version (K_{3j}, $C_{3j} = 0$) of equations (12–15) were programmed, and the "inputs equivalent to potentiometer settings" representing neck stiffness (K_{12}) and damping (C_{12}), body damping (C_{13}), and vertebral damping (C_{11}) were varied until the body to seat acceleration ratio converged to the "shoulder to table" acceleration ratio for a 4 Hz input frequency given by Goldman (1960). The resulting values are shown in Table 1. Using these values, a frequency sweep from 1 to 30 Hz input frequencies showed excellent agreement up to 6 Hz as shown in Figs. 4 and 5 by the curves labeled "linear." A need for nonlinear parameters is clearly indicated.

To determine the nonlinear damping coefficients, the following assumptions were made.

1. All springs are linear ($K_{3j} = 0$).
2. Nonlinear damping in the neck is negligible ($C_{32} = 0$).

With these assumptions, the program for equations

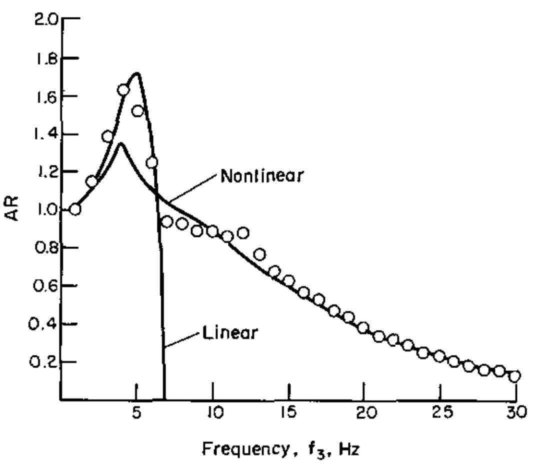

Fig. 4. Body to seat acceleration ratio—AR. ○ Goldman (1960) shoulder to table acceleration ratio.

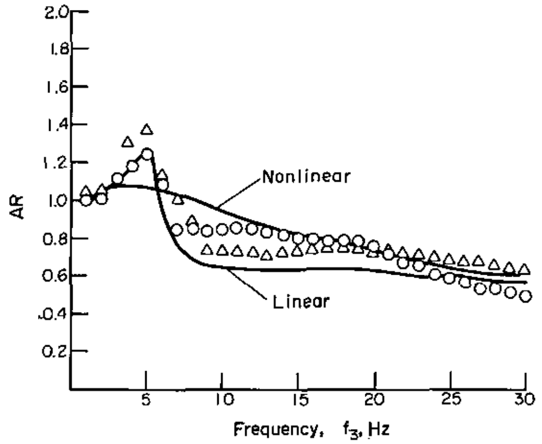

Fig. 5. Head to seat acceleration ratio—AR. ○ Goldman (1960). △ Pradko (1966, 1967).

(12–15) included the previously determined coefficients for the linear terms in addition to the nonlinear terms for vertebral and visceral damping. The procedure for determining the coefficients for the nonlinear terms was similar to that for determining the coefficients for the linear terms. For several input frequencies, the "inputs equivalent to potentiometer settings" representing C_{31} and C_{33} were varied until either the head-to-seat or body-to-seat acceleration ratio converged to the corresponding values given by Goldman (1960). The resulting values are shown in Table 2. The values for C_{33} were graphed. The curve

$$C_{33} = 17289 f_3^2 \tag{16}$$

was found to fit the data reasonably well as shown in Fig. 6. Since the values for C_{31} were very erratic and in all but one case were significantly less than the corresponding value of C_{33}, and since the maximum error was 0.14 for any of the acceleration ratios, it was concluded that nonlinear damping in the vertebral column was not contributory and C_{31} was set to zero in the program, after which a frequency sweep, which included equation (16) as the only coefficient for nonlinear damping, was made for input frequencies ranging from 1 to 30 Hz. These results are shown in Figs. 4 and 5 by the curves labeled "nonlinear". It may be seen that below a 10 Hz input frequency the nonlinear curves give a poor fit to the data. Thus, equation (16) must be modified to

$$C_{33} = 17289 f_3^2 u(f_3 - 10), \tag{17}$$

where

$$u(f_3 - 10) = 0 \; f_3 < 10 \qquad = 1 \; f_3 \geq 10. \tag{18}$$

Table 1. Parameters determined from linear model

No. (j)	K_{1j} (N/m)	C_{1j} (N-sec/m)
1	27158	1780
2	0	686
3	63318	467

Table 2. Parameters determined from nonlinear model

f_3	C_{31}	C_{33}	Steady-State Error |AR| Head/Seat	Body/Seat
6	25462	15403	.02	.14
7	33949	157173	.09	.00
10	11316	2027689	.12	.01
15	5815	4346462	.03	.01
20	1044571	7036478	.00	.00
25	2358	10248780	.08	.00
30	51533	15088608	.12	.01

The units for C_{31} and C_{33} are $N\text{-}(sec/m)^3$.

The responses to input frequencies between 6 and 10 Hz appear to constitute a transition region from linear to nonlinear behavior. Such a region is shown by Magid (1962) and predicted by Muksian and Nash (1974).

CONCLUSIONS

Since time is the only independent variable in the differential equations which describe the motions of the elements, the introduction of frequency as a second independent variable in lumped-parameter models of the human body is difficult to accept because the elements are usually considered to be passive. The results of this preliminary investigation, and the subsequent results (Muksian and Nash, 1974) when related to the physiological responses of vibratory excitations of muscles indicate that when modeling the human body as a lumped-parameter system, the energy absorption elements should be refined to include modeling of the active nature of some of those elements. One such model could be frequency-dependent coefficients for damping (and perhaps stiffness also). Such a phenomenon is considered worthy of further attention by research workers in this active field of biomechanics.

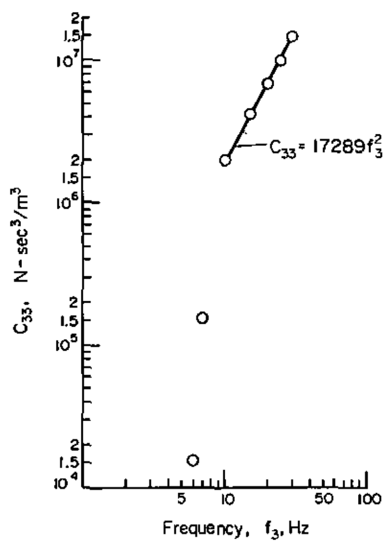

Fig. 6. Frequency dependent damping coefficient. ○ Continuous System Modeling Program convergence calculations.

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NOMENCLATURE

C_{pj}	viscous damping coefficient, $N\text{-}(sec/m)^p$
D_{ji}	viscous damping force on ith mass due to jth dashpot, N
F_i	external force on ith mass except spring and dashpot forces, N
f_i	frequency associated with ith mass, Hz
G	number of body wt.
g	acceleration due to gravity, 9.8 m/sec²
i	subscript representing a particular mass
j	subscript representing a particular spring or dashpot
K_{pj}	spring stiffness, N/m^p
m_i	mass of ith rigid body, kg
p	exponent on relative displacements and relative velocities for nonlinear springs and dashpots, respectively
S_{ji}	spring force on ith mass due to jth spring, N
t	time, sec
z_i	displacement of ith mass, m
$'$	first derivative with respect to time
$''$	second derivative with respect to time
π	3.14159.